

3x3 Application Problems

Find the equation of a parabola in standard form, $y = ax^2 + bx + c$, that passes through the following three points: $(-2, 40)$, $(1, 7)$, and $(3, 15)$.

$$40 = a(-2)^2 + b(-2) + c$$

$$7 = a(1)^2 + b(1) + c$$

$$15 = a(3)^2 + b(3) + c$$

$$4a - 2b + c = 40$$

$$a + b + c = 7$$

$$9a + 3b + c = 15$$

Substitute x and y from each point into the standard form equation to formulate the system

Simplify.

$$a + b + c = 7$$

$$a = -b - c + 7$$

Solve the middle equation for a variable; a in this case

Substitute this equation into the other two equations.

$$4(-b - c + 7) - 2b + c = 40$$

$$-4b - 4c + 28 - 2b + c = 40$$

$$-6b - 3c + 28 = 40$$

$$-6b - 3c = 12$$

$$9(-b - c + 7) + 3b + c = 15$$

$$-9b - 9c + 63 + 3b + c = 15$$

$$-6b - 8c + 63 = 15$$

$$-6b - 8c = -48$$

Solve one of these equation for a variable. Chose the first.

$$-6b - 3c = 12$$

$$-3c = 6b + 12$$

$$c = -2b - 4$$

Substitute this equation into the other to solve for a variable.

$$-6b - 8c = -48$$

$$-6b - 8(-2b - 4) = -48$$

$$-6b + 16b + 32 = -48$$

$$10b + 32 = -48$$

$$10b = -80$$

$$b = -8$$

Substitute this value to find the value of the other variables.

$$c = -2b - 4$$

$$c = -2(-8) - 4$$

$$c = 16 - 4$$

$$c = 12$$

$$a = -b - c + 7$$

$$a = -(-8) - (12) + 7$$

$$a = 8 - 12 + 7$$

$$a = 3$$

The equation of the parabola that goes through the three points is given by: $y = 3x^2 - 8x + 12$