Calculus Section 6.3 Logistic Growth

In exponential growth, we assume that the rate of increase (or decrease) of a population at any time *t* is directly proportional to the population *P*. In other words, . However, in many situations population growth levels off and approaches a limiting number *L* (the carrying capacity) because of limited resources. In this situation the rate of increase (or decrease) is directly proportional to both  This type of growth is called **logistic growth**. It is modeled by the differential equation .   
If we find , we can find out an important fact about the time when *P* is growing the fastest.

**Example)** The population  of fish in a lake satisfies the logistic differential equation , where *t* is measured in years, and .

(a)  (b) What is the range of the solution curve?

(c) For what values of *P* is the solution curve increasing? Decreasing? Justify your answer.

(d) For what values of *P* is the solution curve concave up? Concave down? Justify your answer.

(e) Does the solution curve have an inflection point? Justify your answer.

(f) Use the information you found to sketch the graph of .

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