Solving Absolute Value Inequalities

The way you solve an absolute value inequality depends on the type of inequality symbol.

If the symbol is $<$ or $\leq$, then you use a “double inequality” equation.

Solve: $|2x - 10| \leq 6$

$-6 \leq 2x - 10 \leq 6$

Write as a double inequality

$+10 \qquad +10 \qquad +10$

Add 10 to each side

$\frac{4}{2} \leq \frac{2x}{2} \leq \frac{16}{2}$

Divide each side by 2

$2 \leq x \leq 8$

This says that x is greater than 2 but less than 8.

So, the solution is in between 2 and 8.

$[2, 8]$

Solve: $2|4x + 9| - 3 < 11$

$2|4x + 9| < 14$ First, isolate the abs. value

$|4x + 9| < 7$

$-7 < 4x + 9 < 7$ Write as a double inequality

$-9 \quad -9 \quad -9$ Subtract 9 from each side

$-16 < 4x < -2$

$\frac{-16}{4} < \frac{4x}{4} < \frac{-2}{4}$ Divide each side by 4

$-4 < x < -.5$

This says that x is greater than -4 but less than -.5.

So, the solution is in between -4 and -.5.

(-4, -.5)

If the inequality is > or ≥, then you must solve 2 cases.

Case 1: Simply drop absolute value bars

Case 2: drop abs. value bars, flip the inequality sign, and negate the right side

Solve: $2|x + 3| - 3 > 21$

$2|x + 3| > 24$ Isolate the abs. value

$|x + 3| > 12$

Case 1:

$x + 3 > 12$

$x > 9$

Case 2:

$x + 3 < -12$

$x < -15$

$(-\infty, -15) \cup (9, \infty)$

Solve: $-2|x + 5| - 1 \leq -11$

$-2|x + 5| \leq -10$ Isolate the absolute value.

$|x + 5| \geq 5$

Case 1:

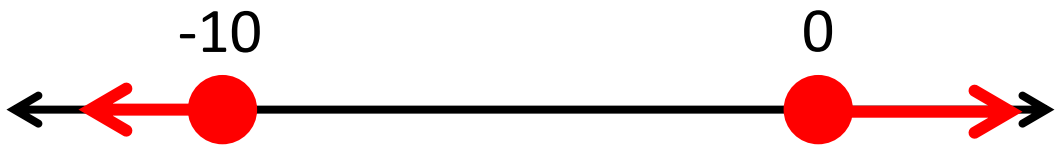
$x + 5 \geq 5$

$x \geq 0$

Case 2:

$x + 5 \leq -5$

$x \leq -10$



$(-\infty, -10] \cup [0, \infty)$