Adding/Subtracting Rational Functions

Warm-Up:

Add/subtract the following (no calculators!):

$$\frac{2}{5} + \frac{7}{15} \qquad\qquad \frac{11}{12} - \frac{3}{8}$$

$$\frac{13}{15} \qquad\qquad\qquad \frac{13}{24}$$

To add/subtract rational functions, you must have a common denominator.

1. Factor

2. Multiply fractions to get a common denom.

3. Combine like terms in the numerator.

$$4 + \frac{x}{x-5} \rightarrow \frac{4}{1} + \frac{x}{x-5}$$

The least common denominator is 1(x – 5)

$$\frac{(x-5)}{(x-5)} \cdot \frac{4}{1} + \frac{x}{x-5}$$

Multiply the 1st fraction by $\frac{x-5}{x-5}$

$$\frac{4x-20}{(x-5)} + \frac{x}{(x-5)}$$

Simplify.

$$\frac{4x-20+x}{(x-5)}$$

Combine fractions.

$$\frac{5x-20}{(x-5)}$$

Combine like terms.

$$\frac{5(x-4)}{(x-5)}$$

Factor to look for holes.

$$\frac{2x-3}{2x-2} - \frac{x-2}{x-1}$$

$$\frac{2x-3}{2(x-1)} - \frac{x-2}{x-1}$$

Factor. The least common denominator is: 2(x −1)

$$\frac{2x-3}{2(x-1)} - \frac{x-2}{x-1} \cdot \frac{2}{2}$$

Multiply the 2nd fraction by $\frac{2}{2}$

$$\frac{2x-3}{2(x-1)} - \frac{2x-4}{2(x-1)}$$

Simplify.

$$\frac{2x-3-2x+4}{2(x-1)}$$

Distribute the subtraction to the numerator and make into one fraction.

$$\frac{1}{2(x-1)}$$

Combine like terms.

$$\frac{2x^2 - 30}{x^2 - 9} - \frac{x + 5}{x + 3}$$

$$\frac{2x^2 - 30}{(x + 3)(x - 3)} - \frac{x + 5}{x + 3}$$ *Factor.*

$$\frac{2x^2 - 30}{(x + 3)(x - 3)} - \frac{x + 5}{x + 3} \cdot \frac{(x - 3)}{(x - 3)}$$ *The least common denominator is: (x + 3)(x − 3)*

$$\frac{2x^2 - 30}{(x + 3)(x - 3)} - \frac{x^2 + 2x - 15}{(x + 3)(x - 3)}$$ *Simplify.*

$$\frac{2x^2 - 30 - x^2 - 2x + 15}{(x + 3)(x - 3)}$$ *Distribute the subtraction to the numerator. Write as one fraction.*

$$\frac{x^2 - 2x - 15}{(x + 3)(x - 3)}$$ *Combine like terms.*

$$\frac{(x-5)(x+3)}{(x+3)(x-3)}$$

Factor the numerator.

$$\frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$$

Cancel common factors.

$$\frac{(x-5)}{(x-3)}$$

$$\frac{2x}{x^2 + 5x + 6} - \frac{3}{x + 1} + \frac{2x}{x + 2}$$

$$\frac{2x}{(x + 3)(x + 2)} - \frac{3}{(x + 1)} + \frac{2x}{(x + 2)}$$

Factor. The least common denominator is: (x+1)(x+2)(x+3)

$$\frac{(x + 1)}{(x + 1)} * \frac{2x}{(x + 3)(x + 2)} - \frac{(x + 3)(x + 2)}{(x + 3)(x + 2)} * \frac{3}{(x + 1)} + \frac{(x + 1)(x + 3)}{(x + 1)(x + 3)} * \frac{2x}{(x + 2)}$$

$$\frac{2x^2 + 2x}{(x + 3)(x + 2)(x + 1)} - \frac{3x^2 + 15x + 18}{(x + 3)(x + 2)(x + 1)} + \frac{2x^3 + 8x^2 + 6x}{(x + 3)(x + 2)(x + 1)}$$

Simplify.

$$\frac{2x^2 + 2x - 3x^2 - 15x - 18 + 2x^3 + 8x^2 + 6x}{(x + 3)(x + 2)(x + 1)}$$

Distribute the negative and write as one fraction.

$$\frac{2x^3 + 7x^2 - 7x - 18}{(x + 3)(x + 2)(x + 1)}$$

Combine like terms. The numerator does not factor, so this is simplified.