Change of Base

If two exponentials have equal bases, then their exponents are also equal.

If $b^x = b^y$, then $x = y$. (Provided $b \neq 0$ and $b \neq 1$)

Ex) $6^{4x-1} = 6^{11}$

$4x - 1 = 11$ *Equal bases means equal exponents*

$x = 3$ *Solve for x.*

Sometimes, you have to re-write your equation so that the bases are the same.

Ex) $3^{2x} = 27$

$(3)^{2x} = (3)^3$	*Rewrite each side with the same base; 3 and 27 are powers of 3.*
$3^{2x} = 3^3$	*To raise a power to a power, multiply exponents.*
$2x = 3$	*Bases are the same, so the exponents must be equal.*
$x = 1.5$	*Solve for x.*

$$9^{8-x} < 27^{x-3}$$

$$(3^2)^{8-x} < (3^3)^{x-3}$$

Rewrite each side with the same base; 9 and 27 are powers of 3.

$$3^{2(8-x)} < 3^{3(x-3)}$$

To raise a power to a power, multiply exponents.

$$16 - 2x < 3x - 9$$

Bases are the same, so the exponents must be equal.

$$25 < 5x$$

Solve for x.

$$5 < x$$

$$x > 5$$

$$4^{x-3} = \frac{1}{64}$$

$$(4)^{x-3} = (4)^{-3}$$

Rewrite each side with the same base; 4 and 64 are powers of 4.

$$x - 3 = -3$$

Bases are the same, so the exponents must be equal.

$$x = 0$$

Solve for x.

$$3^{-2x+1} \cdot 3^{-2x-3} = 9^{-x}$$

$$3^{-2x+1} \cdot 3^{-2x-3} = (3^2)^{-x}$$

Rewrite each side with the same base: 3.

$$3^{-4x-2} = 3^{-2x}$$

Simplify: Power to power = mult. exponents
Multiplied bases = Add the exponents.

$$-4x - 2 = -2x$$

Set the exponents equal to each other and solve.

$$-2 = 2x$$

$$x = -1$$

$$\left(\frac{1}{8}\right)^{2x-1} < 4^{x} \cdot 32^{4-x}$$

$$(2^{-3})^{(2x-1)} < (2^{2})^{(x)} \cdot (2^{5})^{(4-x)}$$

Rewrite each side with the same base: 2.

$$2^{-6x+3} < 2^{2x} \cdot 2^{20-5x}$$

$$2^{-6x+3} < 2^{20-3x}$$

Simplify using exponent properties.

$$-6x + 3 < 20 - 3x$$

Set the exponents equal and solve for x.

$$-17 < 3x$$

$$-5.\overline{6} < x$$

$$x > -5.\overline{6}$$