

Compound and Continuous Interest

Compound Interest is a common way used to collect interest on money.

The diagram shows the compound interest formula $A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$ with several annotations:

- Total amount**: A black arrow points to $A(t)$.
- Principal (initial) amount**: A blue arrow points to P .
- Interest rate**: A red arrow points to r .
- # of times compounded each year**: A green arrow points to n .
- years**: A black arrow points to t .
- nt**: A green arrow points to the exponent nt .

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

Common # of times compounded per year

Name	Times per year
Daily	$n = 365$
Weekly	$n = 52$
Monthly	$n = 12$
Quarterly	$n = 4$
Semi-Annually	$n = 2$
Annually / Yearly	$n = 1$

Simple interest is yearly ($n = 1$) compound interest.

Sally puts \$1,000 into a savings account at her local bank. The saving account has a rate of .45% that is compounded monthly. If Sally leaves the account alone, how much money will be in it after 5 years?

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A(t) = 1000 \left(1 + \frac{.0045}{12} \right)^{12t}$$

$$A(5) = 1000 \left(1 + \frac{.0045}{12} \right)^{12(5)}$$

$$A(5) = \$1022.75$$

Sally puts \$1,000 into a savings account at her local bank. The saving account has a rate of .45% that is compounded daily. If Sally leaves the account alone, how much money will be in it after 5 years?

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A(t) = 1000 \left(1 + \frac{.0045}{365} \right)^{365t}$$

$$A(5) = 1000 \left(1 + \frac{.0045}{365} \right)^{365(5)}$$

$$A(5) = \$1022.75$$

Sally finds a consistent mutual fund that averages a return yield of 8%. Sally invests \$1,000 into the mutual fund and lets it sit for 5 years. Sally's mutual fund compounds quarterly.

$$A(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A(t) = 1000 \left(1 + \frac{.08}{4} \right)^{4t}$$

$$A(5) = 1000 \left(1 + \frac{.08}{4} \right)^{4(5)}$$

$$A(5) = \$1485.95$$

Continuous interest is compound interest compounded every second of every day.

The diagram illustrates the formula for continuous interest: $A(t) = Pe^{rt}$. Each component is labeled with a colored arrow pointing to it:

- Total amount**: A black arrow points to $A(t)$.
- Principal (initial) amount**: A blue arrow points to P .
- Interest rate**: A red arrow points to r .
- years**: A green arrow points to t .
- e, the exponential**: A black arrow points to e , with the text $e \approx 2.718$ below it.

Sally was intrigued by continuous interest. Surely, interest compounded every second of every day would provide for a large return. Sally found a savings account with a .45% rate that compounded continuously. How much money did she have after 5 years?

$$A(t) = Pe^{rt}$$

$$A(t) = 1000e^{0.0045t}$$

$$A(5) = 1000e^{0.0045(5)}$$

$$A(5) = \$1022.76$$