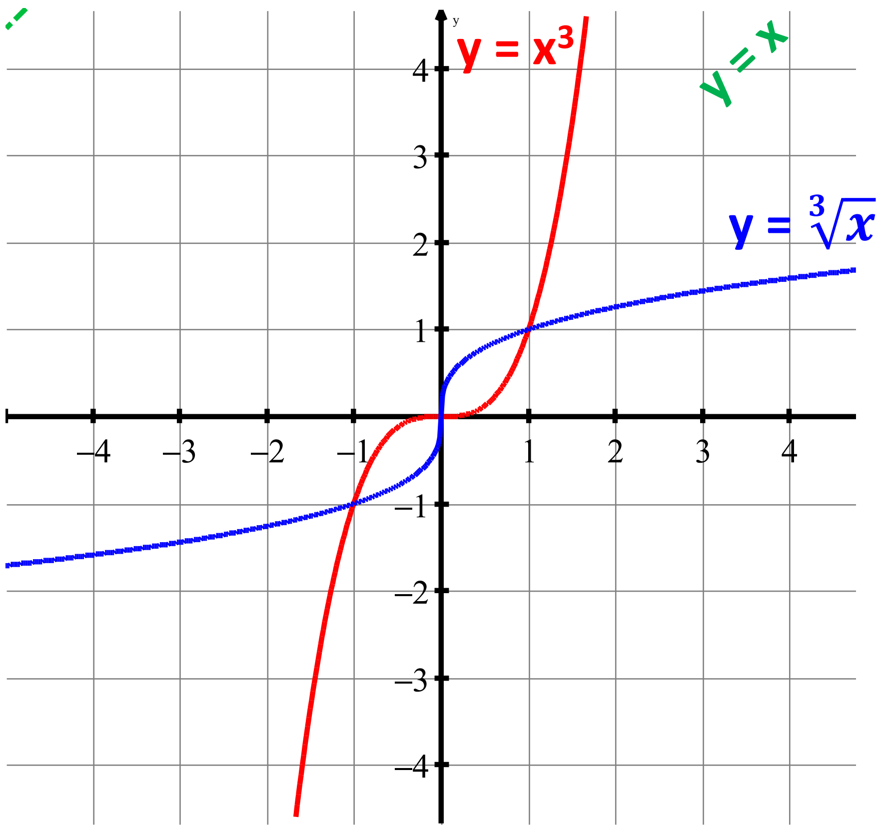
Cubic and Cube Root as Inverses

The cubic function $y = x^3$ and the cube root function $y = \sqrt[3]{x}$ are inverse functions (and inverse operations).



$y = x^3$ and $y = \sqrt[3]{x}$ are reflections over the line y = x.

If $y = x^3$ has the point (2,8), then $y = \sqrt[3]{x}$ has (8,2)

Determine the inverse of g(x) = $\frac{5}{2}(x+1)^3 - 10$.

$$y = \frac{5}{2}(x+1)^3 - 10$$

Write the equation in terms of x and y

$$x = \frac{5}{2}(y+1)^3 - 10$$

Switch the x and y

$$x + 10 = \frac{5}{2}(y+1)^3$$

Solve for y: add 10

$$\frac{2}{5}x + 4 = (y+1)^3$$

Solve for y: multiply by reciprocal of 5/2

$$\sqrt[3]{\frac{2}{5}x + 4 = y + 1}$$

Solve for y: take the cube root of both sides

$$y = \sqrt[3]{\frac{2}{5}x + 4 - 1}$$

Solve for y: subtract 1

Determine the inverse of h(x) = $2\sqrt[3]{2x} + 6$.

$$y = 2\sqrt[3]{2x} + 6$$

Write the equation in terms of x and y

$$x = 2\sqrt[3]{2y} + 6$$

Switch the x and y

$$x - 6 = 2\sqrt[3]{2y}$$

Solve for y: subtract 6

$$\frac{1}{2}x - 3 = \sqrt[3]{2y}$$

Solve for y: divide by 2

$$\left(\frac{1}{2}x - 3\right)^3 = 2y$$

Solve for y: cube both sides

$$y = \frac{\left(\frac{1}{2}x - 3\right)^3}{2}$$

Solve for y: divide by 2

Use composition to show that $f(x) = 2(x-2)^3 + 1$ and $g(x) = \sqrt[3]{\frac{x-1}{2}} + 2$ are inverses.

$$2\left(\sqrt[3]{\frac{x-1}{2}} + 2 - 2\right)^3 + 1$$

Substitute one function into the other.

$$2\left(\sqrt[3]{\frac{x-1}{2}}\right)^3 + 1$$

Cancel out the +2 and -2

$$2\left(\frac{x-1}{2}\right) + 1$$

The third power cancels out the cube root

$$x - 1 + 1$$

Multiplying by 2 cancels out dividing by 2

$$x$$

-1 and +1 cancel each other leaving x. Since the equation simplified to x (and only x), the functions **are inverses**.