Determinant of a Matrix

Square matrices (2x2, 3x3, etc.) have a value called the **determinant**.

The determinant is used when finding the inverse of a matrix.

Determinants may be notated as det(A) or by using straight bars instead of brackets.

Matrix A

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

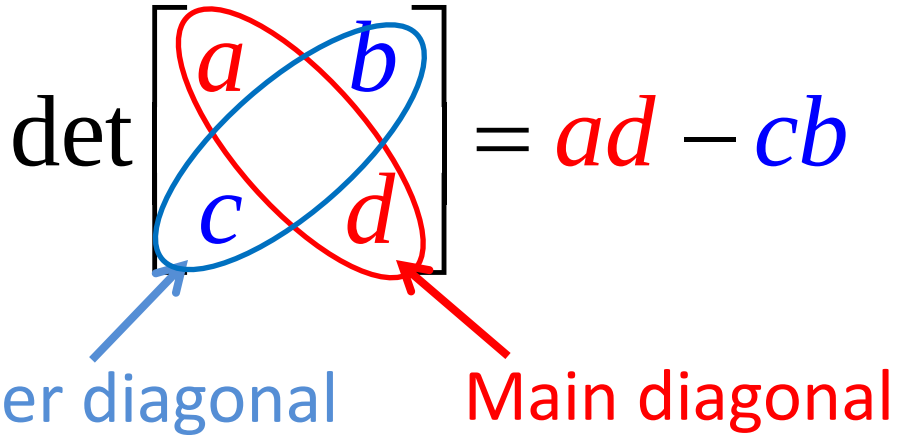
Determinant of A

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

Determinant of a 2x2 Matrix

The determinant is the difference of the product of the diagonals.

Main diagonal – Other diagonal



Find…

$$\begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix} = (1)(8) - (4)(5) = 8 - 20 = -12$$

$$\begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix} = -12$$

Find...

$$\det\begin{bmatrix} 2 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 2 & -3 \\ 4 & 2 \end{vmatrix} = (2)(2) - (4)(-3) = 4 - (-12) = 16$$

$$\det\begin{bmatrix} 2 & -3 \\ 4 & 2 \end{bmatrix} = 16$$

The determinant of matrix A is 4. What is the value of x?

$$A = \begin{bmatrix} 2 & x \\ -3 & -1 \end{bmatrix}$$

$$\begin{vmatrix} 2 & x \\ -3 & -1 \end{vmatrix} = (2)(-1) - (-3)(x) = -2 - (-3x) = 3x - 2$$

$$3x - 2 = 4$$

$$3x = 6$$

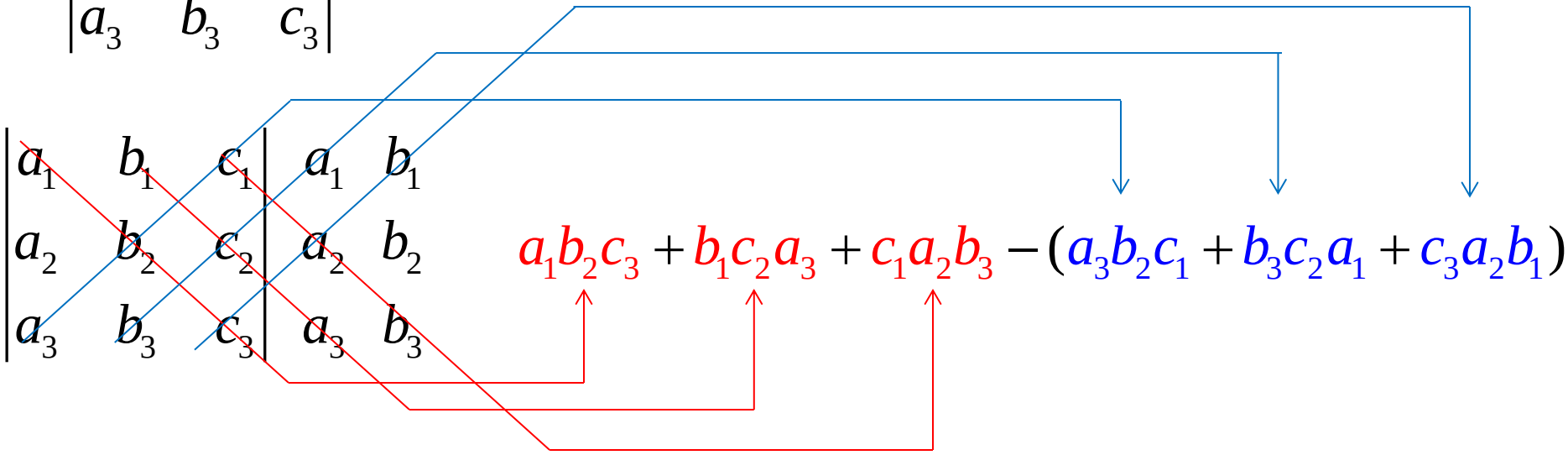
$$x = 2$$

Determinant of a 3x3 Matrix

Rewrite the first two columns to the right of the matrix.

Add the sum of the red diagonals, then subtract the sum of the blue diagonals.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

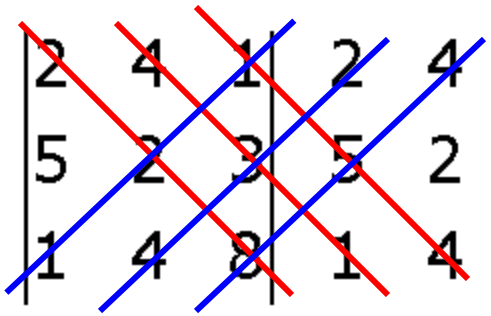


Find the determinant of M.

$$M = \begin{bmatrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{bmatrix} \quad \det M = \begin{vmatrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{vmatrix}, \text{ so write } \begin{vmatrix} 2 & 4 & 1 & 2 & 4 \\ 5 & 2 & 3 & 5 & 2 \\ 1 & 4 & 8 & 1 & 4 \end{vmatrix}$$

Step 1 Multiply each "down" diagonal and add.

$2(2)(8)+ 4(3)(1) + 1(5)(4) = 64$



Step 2 Multiply each "up" diagonal and add.

$1(2)(1)+ 4(3)(2) + 8(5)(4) = 186$

Step 3 Find the difference of the sums.

$64 - 186 = -122$

The determinant is –122.

Application of Determinants

The determinant can be used to find the area of a triangle.

$$Area = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Always report the answer as positive.

Find the area of a triangle with vertices:

(6, 5), (4, -1), (-4, -9).

$$Area = \frac{1}{2}\begin{vmatrix} 6 & 5 & 1 \\ 4 & -1 & 1 \\ -4 & -9 & 1 \end{vmatrix} = \frac{1}{2}(-32) = -16$$

The area of the triangle is 16 units^2.