

Exponential and Logarithmic Properties/Conversion

Exponential functions are written in the form:

$$f(x) = (\textit{number})^x$$

For example, $f(x) = 2^x$, $g(x) = 10^{2x}$, $h(x) = \left(\frac{1}{2}\right)^x$,
 $j(x) = 5^{-x}$, etc.

The number being raised to a power is called the **base**.

A special base for exponentials is “e,” $f(x) = e^x$.

The e stands for (Leonard) Euler’s number.

$e = 2.718\dots$

3 ways to find the value of e:

e is: the value of $\left(1 + \frac{1}{n}\right)^n$ as n approaches infinity.

e is: the value of $(1 + n)^{1/n}$ as n approaches zero.

e is also: $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$

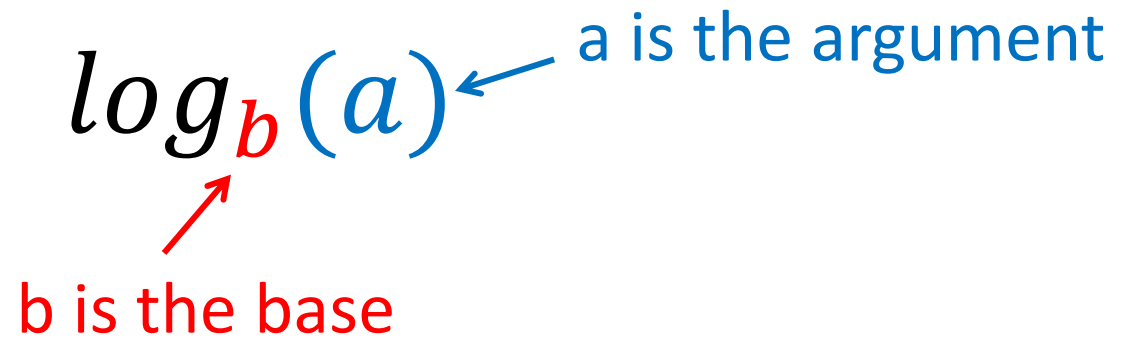
$$\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots \approx 2.718$$

The logarithm (or log) is the inverse for exponentials.

$$\log_b(a)$$

b is the base

a is the argument

The diagram shows the mathematical expression log_b(a). The letter 'b' is written in red, and the letter 'a' is written in blue. A red arrow points from the text 'b is the base' below to the red 'b'. A blue arrow points from the text 'a is the argument' to the right to the blue 'a'.

$$\log_5(3x - 2)$$

5 is the base, $(3x - 2)$ is the argument

This is said, “Log base 5 of $3x - 2$ ”

$\log_2(x)$ is the inverse of 2^x

$\log_{10}(x)$ is the inverse of 10^x

$\log_5(x)$ is the inverse of 5^x

$\ell n(x)$ is the inverse of e^x

$\ell n(x)$ is called the **natural log**. It is the log with a base of e.

A log without a written base is always base 10.

$\log(x)$ implies $\log_{10}(x)$


Typing logarithms into the calculator:

Log base 10: $\log(x)$ use the log button next to 7

Log base e: $\ln(x)$ use the \ln button next to 4

Any other base number: use logBASE from the MATH menu

Logarithmic \leftrightarrow exponential conversion formula:

$$\log_b a = h \leftrightarrow a = b^h$$


The diagram illustrates the conversion of a logarithmic equation to an exponential equation. A blue bracket underlines the base 'b' in the logarithmic equation and the base 'b' in the exponential equation, with the word 'base' written below it.

The base of the log becomes the base of an exponential.

Write $\log_2 32 = x$ as an exponential and solve for x .

$$\log_2 32 = x$$

$$2^x = 32$$

Write the log as an exponential

$$2^5 = 32$$

Solve for x .

$$x = 5$$

Write $\log(x + 8) = 3$ as an exponential and solve for x .

$$\log(x + 8) = 3$$

$$10^3 = x + 8$$

Write the log as an exponential

$$1000 = x + 8$$

Solve for x .

$$x = 992$$

Write each exponential as a log:

a) $5^x = 125$

$$\log_5 125 = x$$

b) $e^5 = x$

$$\ln x = 5$$

c) $x^2 = 25$

$$\log_x 25 = 2$$

d) $25^{1/2} = 5$

$$\log_{25} 5 = 1/2$$

Convert to an exponential equation:

$$w \log_m(x - c) + r = p$$

$$w \log_m(x - c) = p - r \quad \text{Isolate the logarithm first.}$$

$$\log_m(x - c) = \frac{p - r}{w}$$

$$m^{\frac{p-r}{w}} = x - c$$

Convert to an exponential.