Exponential and Logarithmic Properties/Conversion

Exponential functions are written in the form:
$f(x) = (number)^x$

For example, f(x) = 2^x, g(x) = 10^{2x}, h(x) = $\left(\frac{1}{2}\right)^x$,
j(x) = 5^{-x}, etc.

The number being raised to a power is called the **base**.

A special base for exponentials is "e," f(x) = e^x.

The e stands for (Leonard) Euler's number.

e = 2.718….

3 ways to find the value of e:

e is: the value of $\left(1+\frac{1}{n}\right)^n$ as n approaches infinity.

e is: the value of $(1+n)^{1/n}$ as n approaches zero.

e is also: $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots$

$$\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\cdots\approx 2.718$$

The **logarithm** (or log) is the inverse for exponentials.

$$log_b(a)$$

a is the argument

b is the base

$\log_5(3x - 2)$

5 is the base, (3x – 2) is the argument

This is said, "Log base 5 of 3x – 2"

$\log_2(x)$ is the inverse of 2^x

$\log_{10}(x)$ is the inverse of 10^x

$\log_5(x)$ is the inverse of 5^x

$\ell n(x)$ is the inverse of e^x

$\ell n(x)$ is called the **natural log**. It is the log with a base of e.

A log without a written base is always base 10.

$\log(x)$ implies $\log_{10}(x)$

Typing logarithms into the calculator:

Log base 10: log(x) use the log button next to 7

Log base e: $\ell\text{n}(x)$ use the ℓn button next to 4

Any other base number: use logBASE from the MATH menu

Logarithmic ↔ exponential conversion formula:

$$\log_b a = h \quad \leftrightarrow \quad a = b^h$$

base

The base of the log becomes the base of an exponential.

Write $\log_2 32 = x$ as an exponential and solve for x.

$$\log_2 32 = x$$

$$2^x = 32$$

Write the log as an exponential

$$2^5 = 32$$

Solve for x.

$$x = 5$$

Write $\log(x + 8) = 3$ as an exponential and solve for x.

$$\log(x + 8) = 3$$

$$10^3 = x + 8$$

Write the log as an exponential

$$1000 = x + 8$$

Solve for x.

$$x = 992$$

Write each exponential as a log:

a) $5^x = 125$

$$\log_5 125 = x$$

b) $e^5 = x$

$$\ln x = 5$$

c) $x^2 = 25$

$$\log_x 25 = 2$$

d) $25^{1/2} = 5$

$$\log_{25} 5 = 1/2$$

Convert to an exponential equation:

$$w\log_m(x-c)+r=p$$

$$w\log_m(x-c)=p-r$$ Isolate the logarithm first.

$$\log_m(x-c)=\frac{p-r}{w}$$

$$m^{\frac{p-r}{w}}=x-c$$ Convert to an exponential.