Exponential Growth and Decay

Exponential growth occurs when values are increasing. The base will be greater than 1.

Exponential decay occurs when values are decreasing. The base will be between 0 and 1 (or greater than 1 with a negative exponent).

If growing by a rate (doubling, tripling):

$$f(x) = a * b^x$$

Initial amount

Rate of change

Example)
The population of termites in a house triples every month. If there were 20 termites to begin with, how many termites will there be at the end of one year?

$f(x) = a \cdot b^x$

$f(x) = 20 \cdot (3)^x$

$f(x) = 20 \cdot (3)^{12}$

$f(x) = 10{,}628{,}820$ termites

Example)
The mass of a radioactive isotope is halved every year. If a sample of the isotope starts with 75 grams, how long will it take for there to only be 3 grams of the sample?

$f(x) = a \cdot b^x$

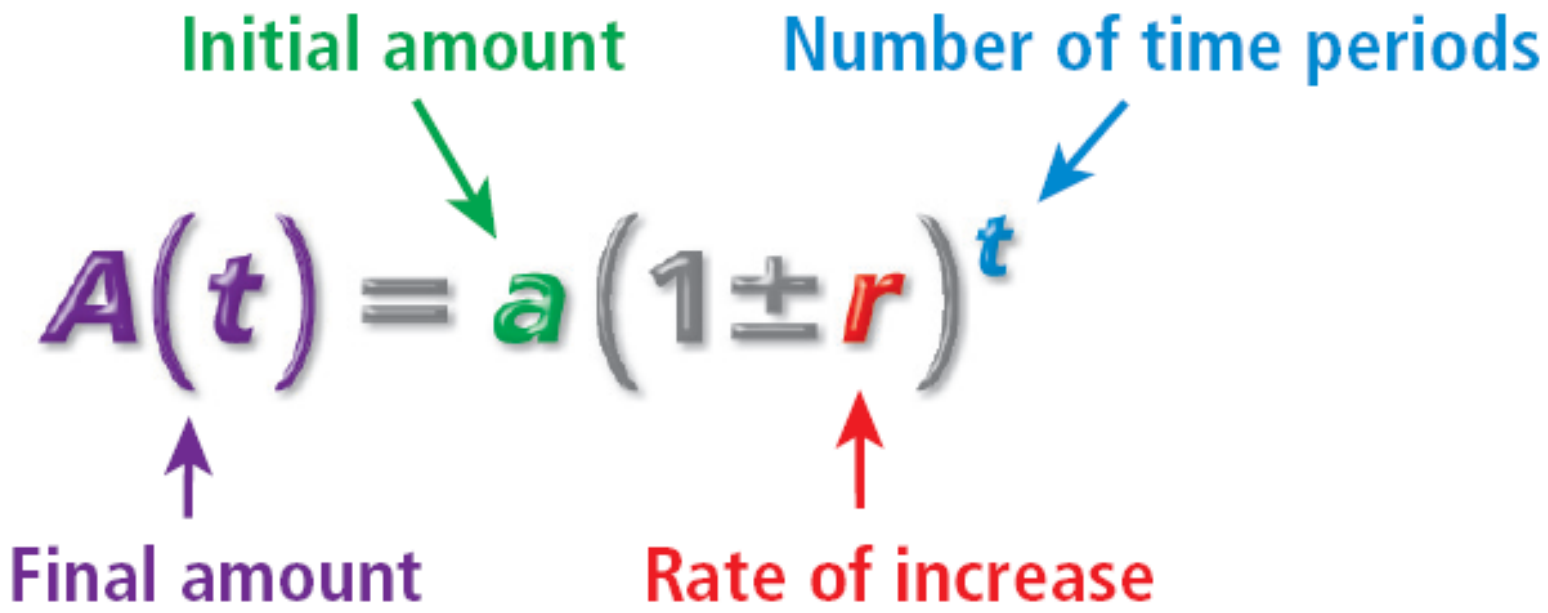
$3 = 75 \cdot \left(\frac{1}{2}\right)^x$

$.04 = \left(\frac{1}{2}\right)^x$

$\log_{1/2}(.04) = x$

x = 4.644 years

If growing/decaying by a <u>percent</u>:



Important: write the rate as a decimal (i.e. 10% = 0.10)

Clara invests $5000 in an account that pays 6.25% interest per year. After how many years will her investment be worth $10,000?

Step 1 Write a function to model the growth in value of her investment.

$A = a(1 + r)^t$ *Exponential growth function.*

$10000 = 5000(1 + 0.0625)^t$ *Substitute 5000 for a, 0.0625 for r, and 10000 for A.*

$10000 = 5000(1.0625)^t$ *Simplify.*

$2 = (1.0625)^t$ *Divide by 5000*

$\log_{1.0625}(2) = t$ *Convert to a log*

$t \approx 11.433$ *Evaluate*

A city population, which was initially 15,500, has been dropping by 3% each year. What is the population of the city after 10 years?

$P(t) = a(1 - r)^t$ *Exponential decay function.*

$P(t) = 15{,}500(1 - 0.03)^{10}$ *Substitute 15,500 for a, 0.03 for r, and 10 for t.*

$P(t) = 15{,}500(0.97)^{10}$ *Simplify.*

$P(t) \approx 11430.07$ *Use your calculator to solve.*

$P(t) \approx 11{,}430$ people *Round to the nearest whole number.*