Exponentials and Logs as Inverses

Because exponentials and logs are inverses:

<u>Exponential</u>	<u>Logarithm</u>
Domain: $(-\infty, \infty)$	Domain: $(0, \infty)$
Range: $(0, \infty)$	Range: $(-\infty, \infty)$

If $y = 3^x$ has the point (2, 9), then $\log_3(x)$ has (9, 2)

The horizontal asymptote of $y = b^x$ is the vertical asymptote of $\log_b(x)$

Find the inverse of f(x) = $\log_5(x+3)-1$ and use composition to prove they are inverses.

Find the inverse:

$x = \log_5(2y+3) - 1$	Switch x and y
$x + 1 = \log_5(2y+3)$	Isolate log: add 1
$5^{x+1} = 2y + 3$	Convert from to an exponential
$5^{x+1} - 3 = 2y$	Isolate y: subtract 3
$\frac{1}{2}(5^{x+1}) - 1.5 = y$	Isolate y: divide by 2
$f^{-1}(x) = \frac{1}{2}(5^{x+1}) - 1.5$	Write using function notation

Use composition to prove they are inverses:

$\log_5((5^{x+1} - 3) + 3) - 1$ Substitute the inverse in place of x

$\log_5(5^{x+1}) - 1$ Simplify: cancel the -3 and +3

$(x+1)\log_5(5) - 1$ Log property: bring down the exponent

$(x+1)(1) - 1$ Log property: if the base and argument are equal, the log = 1

$x + 1 - 1$ Simplify

x The composition simplifies to x, so the functions are inverses.

Find the inverse of g(x) = $2(3)^{2x} + 4$ and use composition to prove they are inverses.

Find the inverse:

$x = 2(3)^{2y} + 4$ Switch x and y

$x - 4 = 2(3)^{2y}$ Isolate the exponential: subtract 4

$\frac{x-4}{2} = (3)^{2y}$ Isolate the exponential: divide by 2

$\log_3\left(\frac{x-4}{2}\right) = 2y$ Convert from exponential to log

$g^{-1}(x) = \frac{1}{2}\log_3\left(\frac{x-4}{2}\right)$ Divide by 2 and write using function notation

Use composition to prove they are inverses:

$$2(3)^{2\left(\frac{1}{2}\log_3\left(\frac{x-4}{2}\right)\right)} + 4$$

Substitute the inverse in place of x

$$2(3)^{\log_3\left(\frac{x-4}{2}\right)} + 4$$

Cancel out the 2 x 1/2

$$2\left(\frac{x-4}{2}\right) + 4$$

3^x and $\log_3(x)$ are inverses, so they cancel each other out

$$(x-4) + 4$$

Cancel out the times 2 and divide by 2

$$x$$

Cancel out -4 and +4

The composition simplifies to x, so the functions are inverses.