Finding and Proving Inverses

To find the inverse of a function, switch the x and y in the equation and solve for y.

Example) $f(x) = 2x - 5$

$y = 2x - 5$	Replace f(x) with y
$x = 2y - 5$	Switch x and y
$x + 5 = 2y$	Solve for y: add 5 to both sides
$\frac{x+5}{2} = y$	Solve for y: divide by 2
$f^{-1}(x) = \frac{1}{2}x + 2.5$	Replace y with the inverse $f^{-1}(x)$

Example) $g(x) = 4(x + 1)^3 + 2$

$y = 4(x + 1)^3 + 2$	Replace $g(x)$ with y
$x = 4(y + 1)^3 + 2$	Switch x and y
$x - 2 = 4(y + 1)^3$	Solve for y: sub. 2 from both sides
$\frac{x-2}{4} = (y + 1)^3$	Solve for y: divide by 4
$\sqrt[3]{\frac{x-2}{4}} = y + 1$	Solve for y: take the cube root of both sides
$\sqrt[3]{\frac{x-2}{4}} - 1 = y$	Solve for y: sub. 1 from both sides
$g^{-1}(x) = \sqrt[3]{\frac{x-2}{4}} - 1$	Replace y with $g^{-1}(x)$

Example) Find the inverse of h(x) = $\sqrt{x-25}+3$

y = $\sqrt{x-25}+3$	Replace h(x) with y
x = $\sqrt{y-25}+3$	Switch x and y
x – 3 = $\sqrt{y-25}$	Solve for y: sub. 3 from both sides
$(x-3)^2 = y - 25$	Solve for y: square both sides
$(x-3)^2 + 25 = y$	Solve for y: add 25 to each side
$h^{-1}(x) = (x-3)^2 + 25$	Replace y with the inverse $h^{-1}(x)$

Two functions can be verified as inverses by substituting them into each other (called composition). This should always simplify to equal x.

Example) f(x) = $\frac{2}{3}x + 6$ and g(x) = $\frac{3}{2}x - 6$

$\frac{2}{3}(\frac{3}{2}x - 6)$ + 6	Substitute g(x) in for x
x – 4 + 6	Simplify: Distribute $\frac{2}{3}$
x + 2	Simplify: combine like terms
$\frac{3}{2}(\frac{2}{3}x + 6)$ - 6	Substitute f(x) in for x
x + 9 - 6	Simplify: Distribute $\frac{3}{2}$
x + 3	Simplify: combine like terms

f(x) and g(x) are <u>not</u> inverses.

Example) Are f(x) = 3x – 1 and g(x) = $\frac{x+1}{3}$ inverses?

$3\left(\frac{x+1}{3}\right) - 1$	Substitute g(x) in for x
x + 1 – 1	Simplify: The 3's cancel
x	Simplify: combine like terms
$\frac{(3x-1)+1}{3}$	Substitute f(x) in for x
$\frac{3x}{3}$	Simplify: Cancel -1 and +1
x	Simplify: Divide 3/3

f(x) and g(x) are inverses.