Graphing Exponentials and Logs

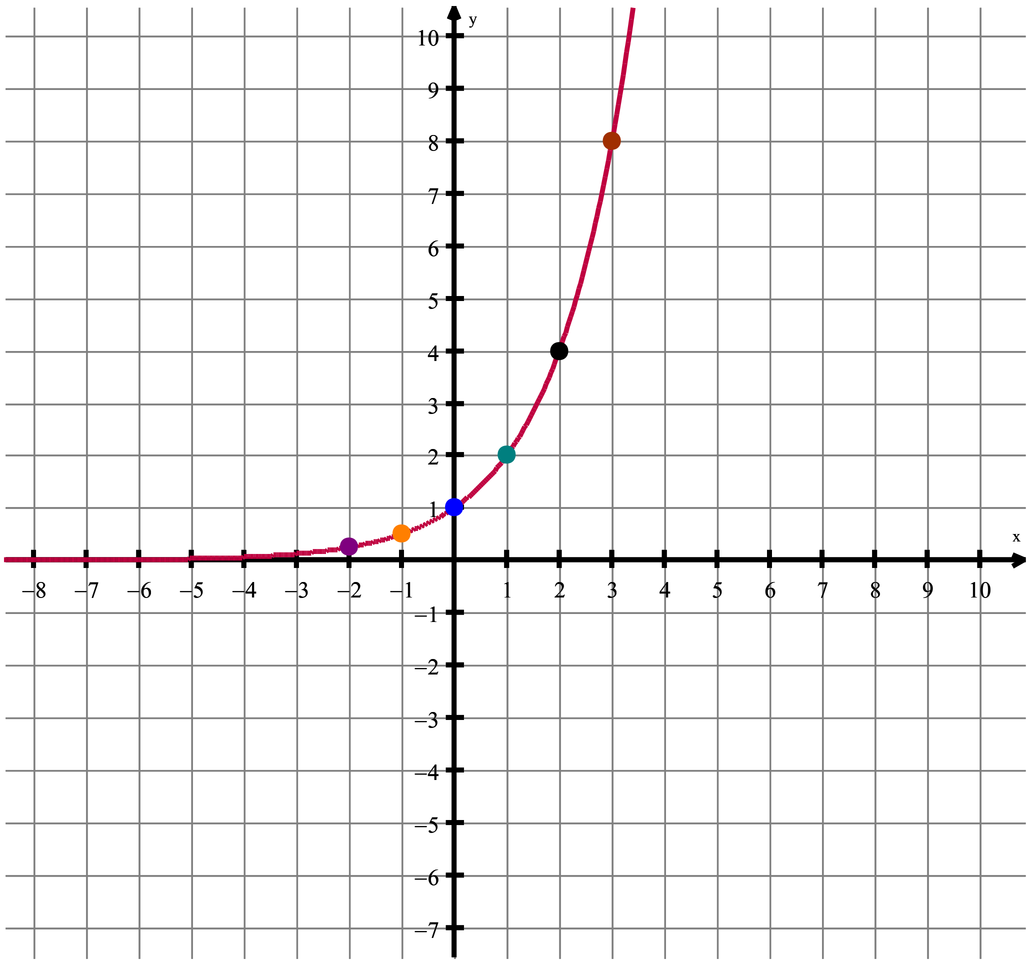
All exponentials (regardless of base)

if there are no transformations will have:

- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- y-intercept: $(0, 1)$ because $(\text{number})^0 = 1$
- x-intercept: None
- Horizontal asymptote: $y = 0$
- End behavior: As $x \to \infty$, $f(x) \to \infty$

  As $x \to -\infty$, $f(x) \to 0$

Graph $f(x) = 2^x$ and identify its properties.



x	f(x)
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$

y-intercept: (0 , 1)

x-intercept: None

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

End behavior:

As $x \to \infty$, $f(x) \to \infty$,

As $x \to -\infty$, $f(x) \to 0$

Transformations of exponentials

Add/subtract in exponent: horizontal shift

$g(x) = b^x + 1$	Vertical shift up 1
$h(x) = b^{x+2}$	Horizontal shift left 2
$j(x) = b^{x-3}$	Horizontal shift right 3
$k(x) = 2b^x$	Vertical stretch by 2
$m(x) = -\frac{1}{3}b^x$	Vertical reflection, vertical compression by 1/3.

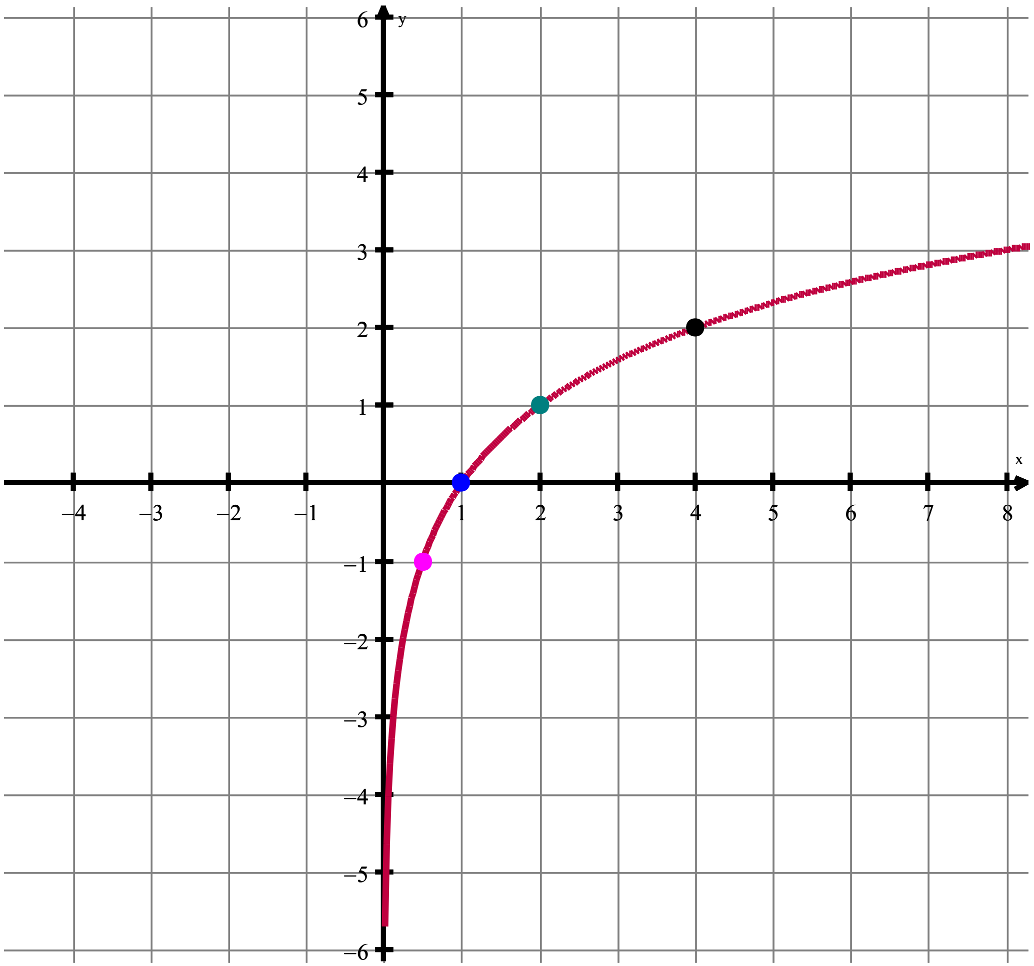
All logarithms (regardless of base)

if there are no transformations will have:

- Domain: $(0, \infty)$
- Range: $(-\infty , \infty)$
- y-intercept: None
- x-intercept: (1, 0) because $\log_b(1) = 0$.
- Vertical asymptote: $x = 0$
- End behavior: As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

  As $x \rightarrow 0^+$, $f(x) \rightarrow -\infty$

Graph $f(x) = \log_2 x$ and identify its properties.



x	f(x)
-1	DNE
0	DNE
1	0
2	1
4	2
8	3

y-intercept: None

x-intercept: (1,0)

Asymptote: x = 0

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

End behavior:

As $x \to 0^+$, $f(x) \to -\infty$,
As $x \to \infty$, $f(x) \to \infty$

Transformations of logs

Add/subtract in argument: horizontal shift

$g(x) = \log_b(x) + 1$	Vertical shift up 1
$h(x) = \log_b(x + 2)$	Horizontal shift left 2
$j(x) = \log_b(x - 3)$	Horizontal shift right 3
$k(x) = 2\log_b(x)$	Vertical stretch by 2
$m(x) = -\frac{1}{3}\log_b(x)$	Vertical reflection, vertical compression by 1/3.

How does the transformation $\log_b(x + 3)$ affect the vertical asymptote of the graph?

The original graph has a vertical asymptote at $x = 0$.

The transformation above is a horizontal shift left 3.

The vertical asymptote of logb(x + 3) is also shifted left 3, so the VA is at $x = -3$.

How does the transformation b^{x-2} affect the asymptote of the graph?

Exponential graphs have horizontal asymptotes.

Shifting the graph right 2 will not affect the asymptote.

A vertical shift would cause the asymptote to change.