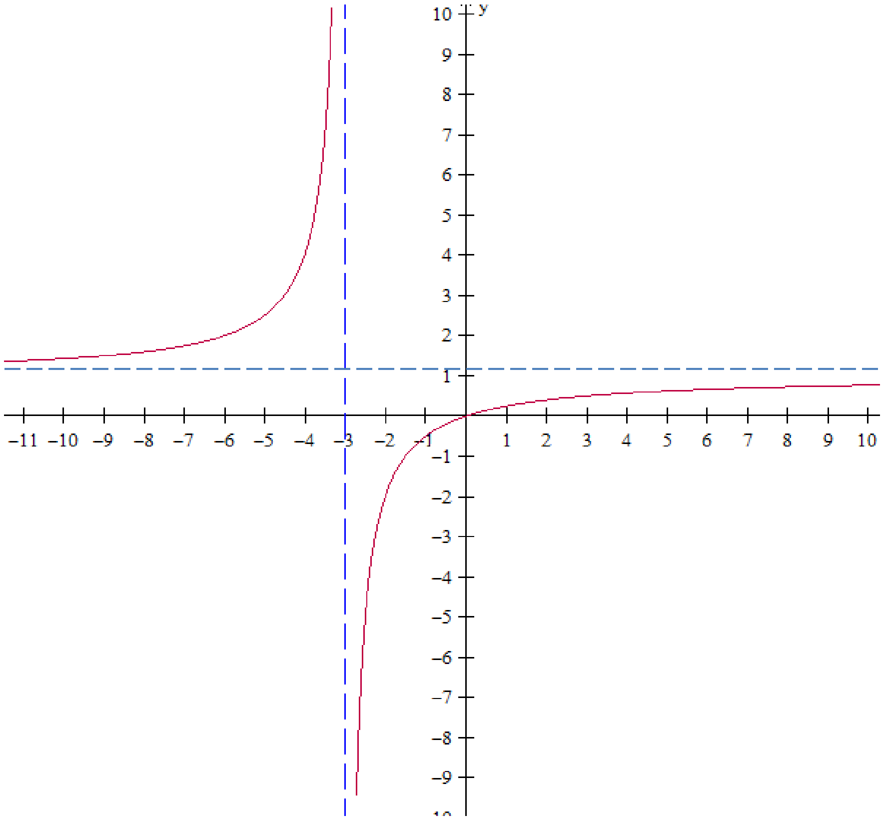
Graphing Rational Functions

Rational functions have variables in the denominator of a fraction (i.e. $\frac{3}{x+2}$, $\frac{x^2+5x+3}{2x+1}$, $\frac{1}{x}$)

An **asymptote** is a line that a graph approaches.



Vertical asymptote at x = -3

Horizontal asymptote at y = 1

Vertical asymptotes occur where the denominator is equal to zero.

Find the vertical asymptotes and domain of:

g(x) = $\frac{x-2}{x^2-1}$

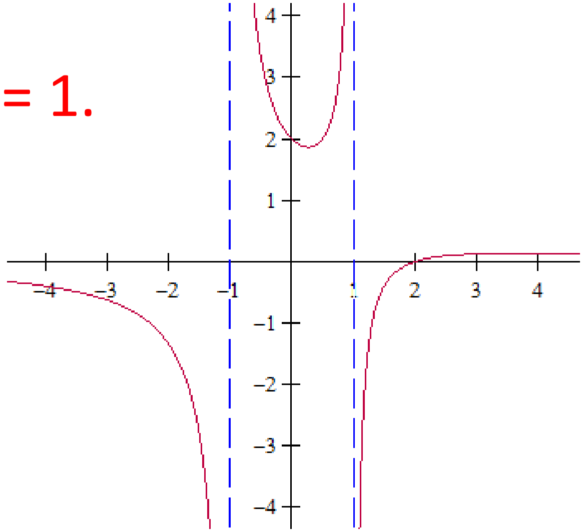
g(x) = $\frac{x-2}{(x+1)(x-1)}$

Factor the denominator

The denominator equals zero when x = -1 and x = 1.

There are vertical asymptotes at x = -1 and x = 1.

The domain is: (-∞, -1)U(-1, 1)U(1, ∞)



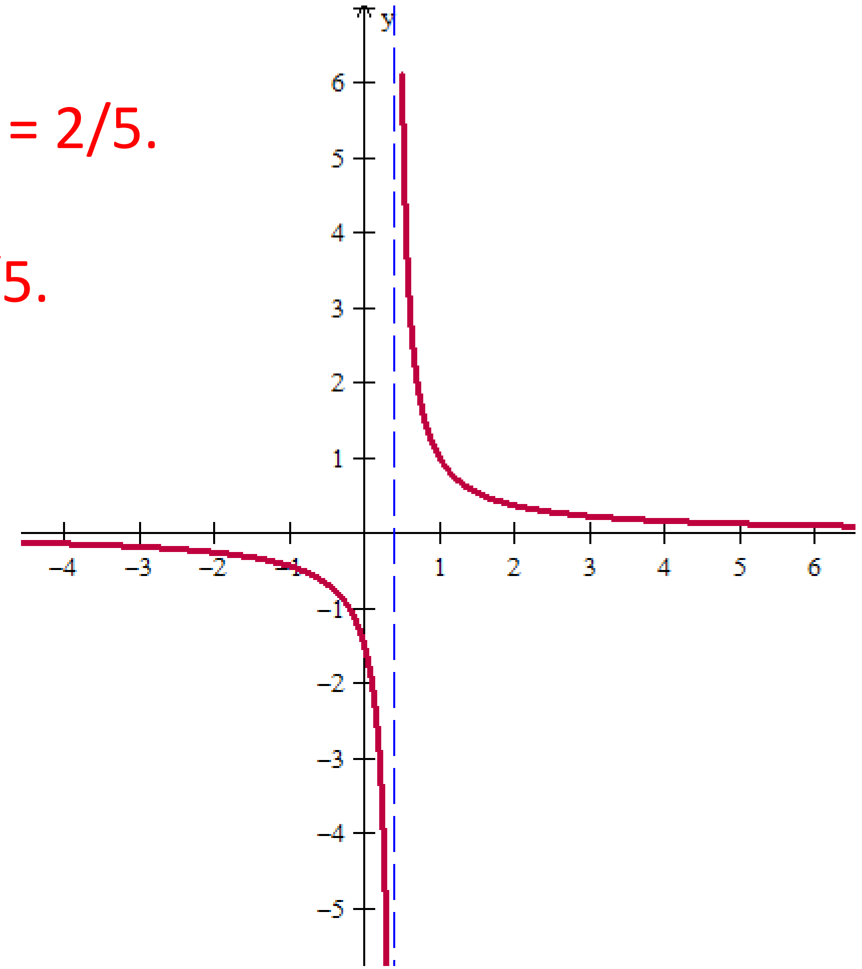
Find the vertical asymptotes and domain for:

$$f(x) = \frac{3}{5x-2}$$

The denominator equals zero when x = 2/5.

There is a vertical asymptote at x = 2/5.

The domain is: (-∞, 2/5)U(2/5, ∞)



A **hole** in a function is a specific x-value where the function does not exist.

Holes are found when you cancel out a factor from the numerator and denominator of the function.

$$h(x) = \frac{(x+1)\cancel{(x+2)}}{x\cancel{(x+2)}}$$

Hole at x = -2

Holes affect the domain (and sometimes range).

Use an open circle to graph a hole.

Determine the holes, vertical asymptotes, and domain for h(x) = $\frac{x^2-7x+6}{x^2-36}$

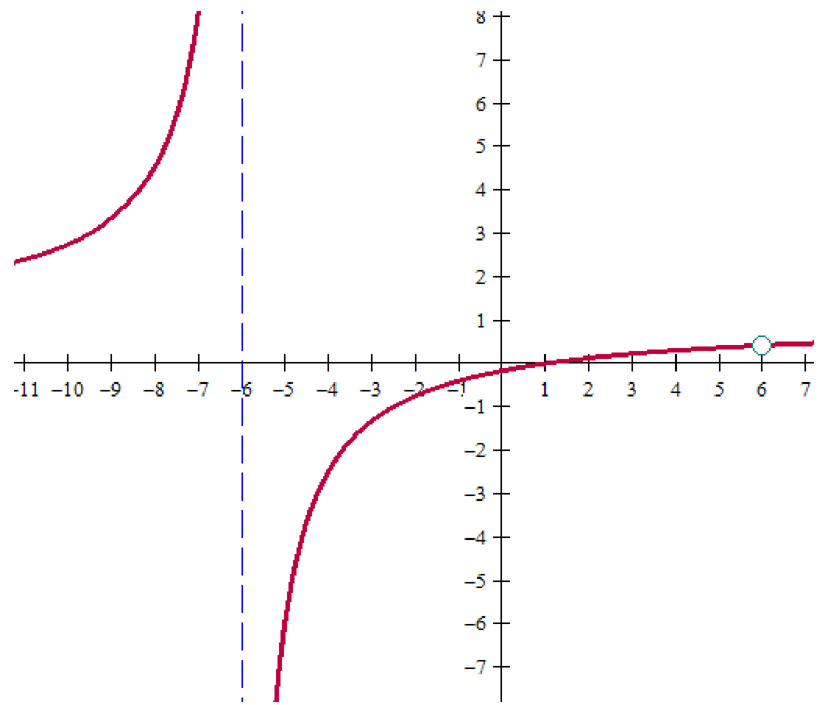
h(x) = $\frac{(x-6)(x-1)}{(x+6)(x-6)}$

Factor the numerator and denominator

h(x) = $\frac{\cancel{(x-6)}(x-1)}{(x+6)\cancel{(x-6)}}$

Cancel common factors

h(x) = $\frac{(x-1)}{(x+6)}$



f(x) has a hole at x = 6 and a vertical asymptote at x = -6

Domain: (-∞, -6)U(-6, 6)U(6, ∞)

Plug in the x-value of a hole into the simplified equation to find its coordinate.

For the last example, the hole is located at x = 6.

$$h(x) = \frac{(x-1)}{(x+6)} \qquad h(6) = \frac{(6-1)}{(6+6)} = \frac{5}{12}$$

The hole has coordinate (6, 5/12)

For horizontal asymptotes (HA):

1. If the numerator has a higher degree, there is no HA.

2. If the denominator has a higher degree, the HA is $y = 0$.

3. If both degrees are the same, the HA is the ratio (fraction) of the leading coefficients.

Determine the horizontal asymptote for:

g(x) = $\frac{3x^2-4x+1}{9x^2+19}$

h(x) = $\frac{5x^5-32x^2}{12x^4-3x}$

The degree of the numerator and denominator are the same.

There is a horizontal asymptote at $\frac{3}{9}=\frac{1}{3}$, y = $\frac{1}{3}$

The degree of the numerator is greater than the denominator.

The function has no horizontal asymptote.

Determine the horizontal asymptote for:

g(x) = $\frac{x^2+2x+2}{2x^3+5}$

The degree of the denominator is greater than the numerator.

There is a horizontal asymptote at y = 0.