Dividing Polynomials: Long Division

Warm-Up: Use long division to evaluate each quotient by hand (no calculator allowed)

$6\overline{)214}$

$$\begin{array}{r} 35 \\ 6\overline{)214} \\ -18 \\ \hline 34 \\ -30 \\ \hline 4 \end{array}$$

4 ← Remainder

$214 \div 6 = 35\frac{4}{6}$

$12\overline{)277}$

$$\begin{array}{r} 23 \\ 12\overline{)277} \\ -24 \\ \hline 37 \\ -36 \\ \hline 1 \end{array}$$

1 ← Remainder

$277 \div 12 = 23\frac{1}{12}$

Divide using long division.
$(-y^2 + 2y^3 - 45) \div (y - 3)$

Step 1 Write the dividend in standard form, including terms with a coefficient of 0.

$2y^3 - y^2 + 0y - 45$

Step 2 Write division in the same way you would when dividing numbers.

$$y - 3 \overline{)2y^3 - y^2 + 0y - 45}$$

Step 3 Divide.

$$y - 3 \overline{)\, 2y^3 - y^2 + 0y - 45}$$

$$\begin{array}{r} 2y^2 \\ \hline 2y^3 - y^2 + 0y - 45 \\ -(2y^3 - 6y^2) \\ \hline 5y^2 \end{array}$$

What multiplies with y to equal $2y^3$? $2y^2$. Write $2y^2$ above the division line.

Multiply $y - 3$ by $2y^2$. Then subtract those values divided polynomial.

Step 3 Divide.

$$
\begin{array}{r}
2y^2 + 5y \\
y - 3 \overline{)\,2y^3 - y^2 + 0y - 45} \\
-(2y^3 - 6y^2) \\
\hline
5y^2 + 0y \\
-(5y^2 - 15y) \\
\hline
15y
\end{array}
$$

Bring down the next term (0y).

What multiplies by y to equal $5y^2$? 5y

Multiply y – 3 by 5y. Then subtract.

Step 3 Divide.

$$
\begin{array}{r}
2y^2+5y+15 \\
y-3\overline{)\,2y^3-y^2+0y-45} \\
-(2y^3-6y^2) \\
\hline
5y^2+0y \\
-(5y^2-15y) \\
\hline
15y-45 \\
-(15y-45) \\
\hline
0
\end{array}
$$

Bring down the next term (45). What multiplies with y to equal 15y? 15

Multiply y – 3 by 15. Then subtract.

The remainder is 0.

$2y^2 + 5y + 15$

The answer is what is above the division symbol.

You can check your answer by multiplying $(y – 3)(2y^2 + 5y + 15)$

Divide using long division.
$(15x^2 + 8x - 12) \div (3x + 1)$

Step 1 Write the dividend in standard form, including terms with a coefficient of 0.

$15x^2 + 8x - 12$

Step 2 Write division in the same way you would when dividing numbers.

$$3x + 1 \overline{)\, 15x^2 + 8x - 12}$$

$$
\begin{array}{r}
5x + 1 \\
3x + 1 \overline{)\,15x^2 + 8x - 12} \\
-(15x^2 + 5x) \\
\hline
3x - 12 \\
-(3x + 1) \\
\hline
-13
\end{array}
$$

Notice that 3x times 5x is $15x^2$. Write 5x above $15x^2$.

Multiply 3x + 1 by 5x. Then subtract. Bring down the next term. Divide 3x by 3x.

Multiply 3x + 1 by 1. Then subtract.

Find the remainder. It goes over the divisor.

$$5x + 1 - \frac{13}{3x + 1}$$

Divide using long division.

$(4x^3 - 2x^2 - 3) \div (2x^2 - 1)$

Step 1 Write the dividend in standard form, including terms with a coefficient of 0.

$4x^3 - 2x^2 + 0x - 3$

Step 2 Write division in the same way you would when dividing numbers.

$$2x^2 - 1 \overline{)4x^3 - 2x^2 + 0x - 3}$$

$$
\begin{array}{r}
2x - 1 \\
2x^2 - 1 \overline{)\,4x^3 - 2x^2 + 0x - 3} \\
\underline{-(4x^3 + \ 0x^2 - 2x)} \\
-2x^2 + 2x - 3 \\
\underline{-(-2x^2 + 0x + 1)} \\
2x - 4
\end{array}
$$

Notice that 3x times 5x is $15x^2$. Write 5x above $15x^2$.

Multiply 3x + 1 by 5x. Then subtract. Bring down the next term. Divide 3x by 3x.

Multiply 3x + 1 by 1. Then subtract.

Find the remainder.

$$2x - 1 - \frac{2x - 4}{2x^2 - 1}$$

You are finished with long division once what you are dividing into has a lower power than what you are dividing by.

The last problem was finished once we divided down to $(2x - 4)$ because $2x$ has a lower degree than the divisor: $2x^2 - 1$.

Evaluate $(x^4 + 3x^3 + 5x - 1) \div (x^2 - 2)$

$$
\begin{array}{r}
x^2 + 3x + 2 \\
x^2 + 0x - 2 \overline{)\, x^4 + 3x^3 + 0x^2 + 5x - 1} \\
-(x^4 + 0x^3 - 2x^2) \\
\hline
3x^3 + 2x^2 + 5x \\
-(3x^3 + 0x^2 - 6x) \\
\hline
2x^2 + 11x - 1 \\
-(2x^2 + 0x - 4) \\
\hline
11x + 3
\end{array}
$$

$$x^2 + 3x + 2 + \frac{11x + 3}{x^2 - 2}$$