Matrix Basics

A **matrix** (plural: matrices) is used to organize data or systems of equations.

Matrices have rows (horizontal) and columns (vertical).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Row 1
Row 2

Column 1 Column 2 Column 3

The **dimensions** of a matrix come from the number of rows and columns. Always rows 1^{st} and columns 2^{nd}.

Matrix A is a 2x3 (2 rows by 3 columns) matrix.

What are the dimensions of matrix B and matrix C?

$$B = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \\ 8 & 1 \end{bmatrix} \qquad \mathrm{C} = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 5 & -2 \end{bmatrix}$$

B is 4x2 and C is 2x3

B_{4x2} C_{2x3}

The dimensions are written as subscripts.

The **address** (location) of an entry is based on the row and column.

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

2^{nd} row, 1^{st} column

Matrices are named with capital letters.

Addresses are named with lowercase letters.

Two matrices can be added/subtracted if they have the exact same dimensions.

Add or subtract, if possible.

$$\mathbf{W} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

W + Y

Add each corresponding entry.

$$W + Y = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 + 1 & -2 + 4 \\ 1 + (-2) & 0 + 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$$

Add or subtract, if possible.

$$\mathbf{X} = \begin{bmatrix} 4 & 7 & c \\ a & 1 & -1 \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} b & -2 & 3 \\ 1 & 0 & 4 \end{bmatrix}$$

X – Z

Subtract each corresponding entry.

$$X - Z = \begin{bmatrix} 4 & 7 & c \\ a & 1 & -1 \end{bmatrix} - \begin{bmatrix} b & -2 & 3 \\ 1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 4\text{-}b & 9 & c\text{-}3 \\ a\text{-}1 & 1 & -5 \end{bmatrix}$$

Add or subtract if possible.

$$\mathbf{A} = \begin{bmatrix} 4 & -2 \\ -3 & 10 \\ 2 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & -1 & -5 \\ 3 & 2 & 8 \end{bmatrix}$$

B – A

B is a 2×3 matrix, and A is a 3×2 matrix. Because B and A do not have the same dimensions, they cannot be subtracted.

Matrices can be multiplied by a constant number called a **scalar**. Multiply all entries by the scalar.

$$\mathbf{A} = \begin{bmatrix} 4 & -2 \\ -3 & 10 \end{bmatrix}$$

Determine the value of -3A.

$$-3\begin{bmatrix} 4 & -2 \\ -3 & 10 \end{bmatrix} \longrightarrow \begin{bmatrix} 4(-3) & -2(-3) \\ -3(-3) & 10(-3) \end{bmatrix} \longrightarrow -3\mathbf{A} = \begin{bmatrix} -12 & 6 \\ 9 & -30 \end{bmatrix}$$

Given: $A = \begin{bmatrix} 3 & x+4 \\ y & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & -2x \\ 3 & 0 \end{bmatrix}$, determine 2A – 3B.

$$2\begin{bmatrix} 3 & x+4 \\ y & -2 \end{bmatrix} - 3\begin{bmatrix} 9 & -2x \\ 3 & 0 \end{bmatrix}$$

Write out the problem.

$$\begin{bmatrix} 6 & 2x+8 \\ 2y & -4 \end{bmatrix} + \begin{bmatrix} -27 & 6x \\ -9 & 0 \end{bmatrix}$$

Multiply by each scalar.

$$\begin{bmatrix} -21 & 8x+8 \\ 2y-9 & -4 \end{bmatrix}$$

Combine corresponding entries.

Two matrices are equal if and only if they have the same dimensions and equal entries.

$$A = \begin{bmatrix} 4 & 0 \\ 6 & -2 \\ 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} x & 0 \\ 6 & y+4 \\ z-x & 1 \end{bmatrix}$$

If A = B, determine the value of each variable.

$$\begin{bmatrix} 4 & 0 \\ 6 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 6 & y+4 \\ z-x & 1 \end{bmatrix}$$

x = 4

y + 4 = -2 → y = -6

z – x = 3 → z = x + 3 → z = 7