Multiplying/Dividing Rational Functions

Warm-up

Multiply or divide the following (no calculators!):

1) $\dfrac{3}{4} \times \dfrac{3}{5}$ 　　　　　2) $\dfrac{x}{4} \times \dfrac{2x}{7}$ 　　　　　3) $\dfrac{5}{7} \div \dfrac{3}{2}$

When multiplying rational functions:

1) Factor first

2) Top x Top, Bottom x Bottom

3) Cancel out common factors (holes)

When dividing rational functions:

1) ***Flip the dividing function***

2) Multiply

Multiply.

$$\frac{3-x}{4x+20} \cdot \frac{x+5}{x^2-9}$$

$$\frac{-1(x-3)}{4(x+5)} \cdot \frac{x+5}{(x-3)(x+3)}$$ *Factor each rational*

$$\frac{-1}{4(x+3)}$$ *Cancel common factors*

Multiply.

$$\frac{10x - 40}{x^2 - 6x + 8} \bullet \frac{x - 4}{5x - 10}$$

$$\frac{10(x - 4)}{(x - 4)(x - 2)} \bullet \frac{(x - 4)}{5(x - 2)}$$ *Factor each rational*

$$\frac{2(x - 4)}{(x - 2)^2}$$ *Cancel common factors*

Divide.

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \div \frac{x^4 + 2x^3 - 8x^2}{x^2 - 16}$$

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \cdot \frac{x^2 - 16}{x^4 + 2x^3 - 8x^2}$$

Rewrite as multiplication by the reciprocal.

$$\frac{x^2(x^2 - 9)}{x^2 - 4x + 3} \cdot \frac{x^2 - 16}{x^2(x^2 + 2x - 8)}$$

Factor.

$$\frac{\cancel{x^2}\cancel{(x - 3)}(x + 3)}{\cancel{(x - 3)}(x - 1)} \cdot \frac{\cancel{(x + 4)}(x - 4)}{\cancel{x^2}(x - 2)\cancel{(x + 4)}}$$

Cancel common factors.

$$\frac{(x + 3)(x - 4)}{(x - 1)(x - 2)}$$

Evaluate.

$$\frac{\dfrac{2x^2 - 7x - 4}{x^2 - 9}}{\dfrac{4x^2 - 1}{8x^2 - 28x + 12}}$$

$$\frac{2x^2 - 7x - 4}{x^2 - 9} \bullet \frac{8x^2 - 28x + 12}{4x^2 - 1}$$

Rewrite as multiplication by the reciprocal.

$$\frac{(2x + 1)(x - 4)}{(x + 3)(x - 3)} \bullet \frac{4(2x^2 - 7x + 3)}{(2x + 1)(2x - 1)}$$

Factor.

$$\frac{\cancel{(2x + 1)}(x - 4)}{(x + 3)\cancel{(x - 3)}} \bullet \frac{4\cancel{(2x - 1)}\cancel{(x - 3)}}{\cancel{(2x + 1)}\cancel{(2x - 1)}}$$

Cancel common factors.

$$\frac{4(x - 4)}{(x + 3)}$$

Evaluate.

$$\dfrac{\dfrac{x^2 + 10x - 24}{x^2 - 4x}}{x^2 + 2x - 8} = \dfrac{\dfrac{x^2 + 10x - 24}{x^2 - 4x}}{\dfrac{x^2 + 2x - 8}{1}}$$

$$\dfrac{x^2 + 10x - 24}{x^2 - 4x} \bullet \dfrac{1}{x^2 + 2x - 8}$$ *Rewrite as multiplication by the reciprocal.*

$$\dfrac{(x + 12)(x - 2)}{(x)(x - 4)} \bullet \dfrac{1}{(x + 4)(x - 2)}$$ *Factor.*

$$\dfrac{(x + 12)\cancel{(x - 2)}}{(x)(x - 4)} \bullet \dfrac{1}{(x + 4)\cancel{(x - 2)}}$$ *Cancel common factors.*

$$\dfrac{(x + 12)}{x(x - 4)(x + 4)}$$

Simplify.

$$\frac{x^2 - 1}{x^2 + 5x - 6} \div \frac{x^2 + x}{x^2 + 5x + 6} \cdot \frac{3 - x}{x^2 + 4x + 4}$$

$$\frac{x^2 - 1}{x^2 + 5x - 6} \cdot \frac{x^2 + 5x + 6}{x^2 + x} \cdot \frac{3 - x}{x^2 + 4x + 4}$$

$$\frac{(x + 1)(x - 1)}{(x + 6)(x - 1)} \cdot \frac{(x + 2)(x + 3)}{x(x + 1)} \cdot \frac{-(x - 3)}{(x + 2)(x + 2)}$$

$$\frac{(x + 1)(x - 1)}{(x + 6)(x - 1)} \cdot \frac{(x + 2)(x + 3)}{x(x + 1)} \cdot \frac{-(x - 3)}{(x + 2)(x + 2)}$$

$$\frac{-(x + 3)(x - 3)}{(x)(x + 6)(x + 2)}$$