Multiplying Matrices

Two matrices can be multiplied if:

Matrix 1's columns = Matrix 2's rows

Given A_{4x2} and B_{2x5}

A B

AB is possible because: 4 x 2 2 x 5

The resulting matrix is 4x5 (the outside numbers).

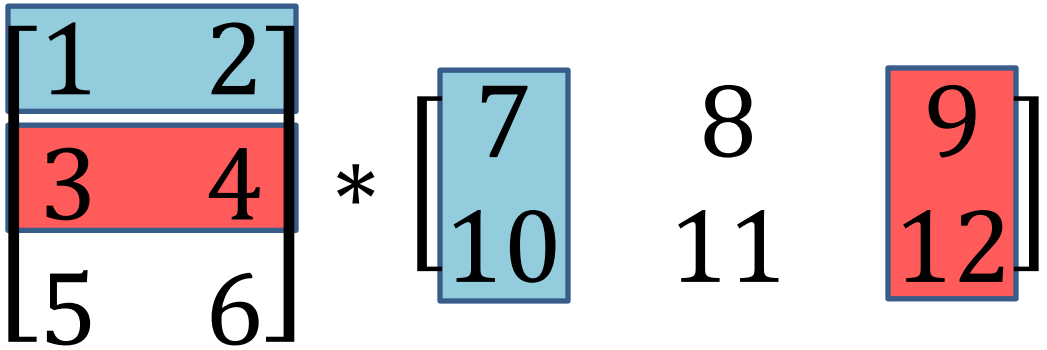
B A

BA is <u>not</u> possible because: 2 x 5 4 x 2

The inside numbers do not match.

To multiply matrices:

Multiply each entry from a row of the 1st matrix with each entry from a column of the 2nd.



1st row times 1st column gives address a_{11}: 1(7) + 2(10) = 27

2nd row times 3rd column gives address a_{23}: 3(9) + 4(12) = 75

$$\begin{bmatrix} 27 & . & . \\ . & . & 75 \\ . & . & . \end{bmatrix}$$

Find the product, if possible.

WX

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Check the dimensions. W is 3×2 , X is 2×3 .
WX is defined and is 3×3.

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 1 of *W* and column 1 of *X* as shown. Place the result in wx_{11}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$3(4) + -2(5)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 1 of *W* and column 2 of *X* as shown. Place the result in wx_{12}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$3(7) + -2(1)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 1 of W and column 3 of X as shown. Place the result in wx_{13}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$3(-2) + -2(-1)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 2 of W and column 1 of X as shown. Place the result in wx_{21}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$1(4) + 0(5)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 2 of *W* and column 2 of *X* as shown. Place the result in wx_{22}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & ? \\ ? & ? & ? \end{bmatrix}$$

$1(7) + 0(1)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 2 of W and column 3 of X as shown. Place the result in wx_{23}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ ? & ? & ? \end{bmatrix}$$

$1(-2) + 0(-1)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 3 of *W* and column 1 of *X* as shown. Place the result in wx_{31}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & ? & ? \end{bmatrix}$$

$2(4) + -1(5)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 3 of *W* and column 2 of *X* as shown. Place the result in wx_{32}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & 13 & ? \end{bmatrix}$$

$2(7) + -1(1)$

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Multiply row 3 of *W* and column 3 of *X* as shown. Place the result in wx_{33}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & 13 & -3 \end{bmatrix}$$

$2(-2) + -1(-1)$

Find each product, if possible.

XW

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix}$$

Check the dimensions. X is 2×3, and W is 3×2 so the product is defined and is 2×2.

$$XW = \begin{bmatrix} 4(3)+7(1)-2(2) & 4(-2)+7(0)-2(-1) \\ 5(3)+1(1)-1(2) & 5(-2)+1(0)-1(-1) \end{bmatrix} = \begin{bmatrix} 15 & -6 \\ 14 & -9 \end{bmatrix}$$

Find the product, if possible.

CA

$$A = \begin{bmatrix} 0 & 4 & 9 \\ -3 & 3 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 11 & -1 \\ 12 & 10 \end{bmatrix}$$

Check the dimensions. C is 2×2, and A is 2×3 so the product is defined and is 2×3.

$$CA = \begin{bmatrix} 11(0) - 1(-3) & 11(4) - 1(3) & 11(9) - 1(2) \\ 12(0) + 10(-3) & 12(4) + 10(3) & 12(9) + 10(2) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 41 & 97 \\ -30 & 78 & 128 \end{bmatrix}$$

John and Paul both take a multiple choice test. Their results are shown in the table below.

	# Correct	# No Answer	# Incorrect
John	18	7	5
Paul	20	0	10

Correct answers = 2 pts

No answer = 0 pts

Incorrect answer = -1 pt

How many points did John and Paul score?

Set up two matrices: one for their answers and one from the points awarded.

$$P = \begin{matrix} \text{Correct} & \text{No attempt} & \text{Incorrect} \\ \begin{bmatrix} 18 & 7 & 5 \\ 20 & 0 & 10 \end{bmatrix} & & \end{matrix} \begin{matrix} \\ \textit{John} \\ \textit{Paul} \end{matrix}$$

$$Q = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \begin{matrix} \textit{Correct} \\ \textit{No attempt} \\ \textit{Incorrect} \end{matrix}$$

Multiply the two matrices.

$$\begin{bmatrix} 18 & 5 & 7 \\ 20 & 0 & 10 \end{bmatrix} * \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 18(2)+7(0)+5(-1) \\ 20(2)+0(0)+10(-1) \end{bmatrix} = \begin{bmatrix} 31 \\ 30 \end{bmatrix}$$

John scored 31 points and Paul scored 30 points.