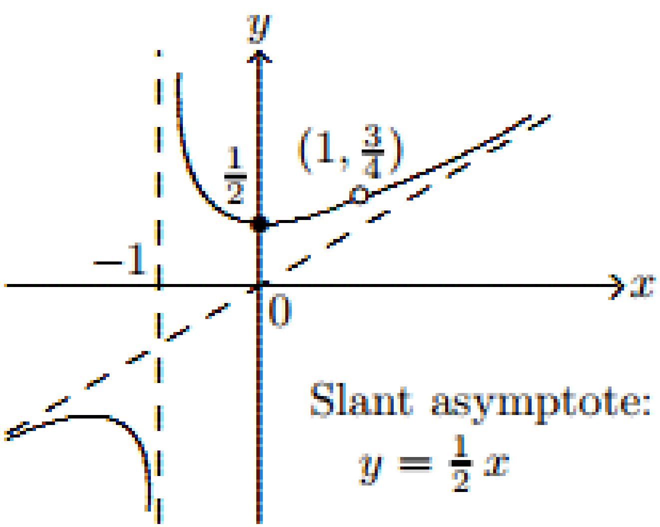
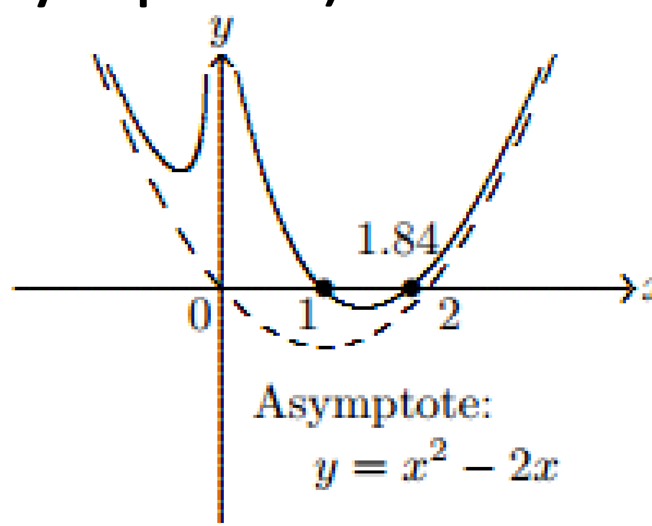
Oblique (Slant) Asymptotes

An **oblique asymptote** is a slanted (diagonal) or curved asymptote.

Oblique asymptotes occur when the degree of the numerator is greater than the denominator (that's why there isn't a horizontal asymptote).





If the degree of the numerator is 1 more than the denominator ($\frac{x^4}{x^3}$), then the asymptote will be linear.

If the degree of the numerator is 2 more than the denominator ($\frac{x^5}{x^3}$), then the asymptote will be a quadratic function. 3 more is cubic, etc.

We will focus solely on linear oblique asymptotes.

Graph the equation $\mathrm{f}(\mathrm{x}) = \frac{2x^2}{1-x}$ and determine its attributes.

Vertical Asymptote: x = 1

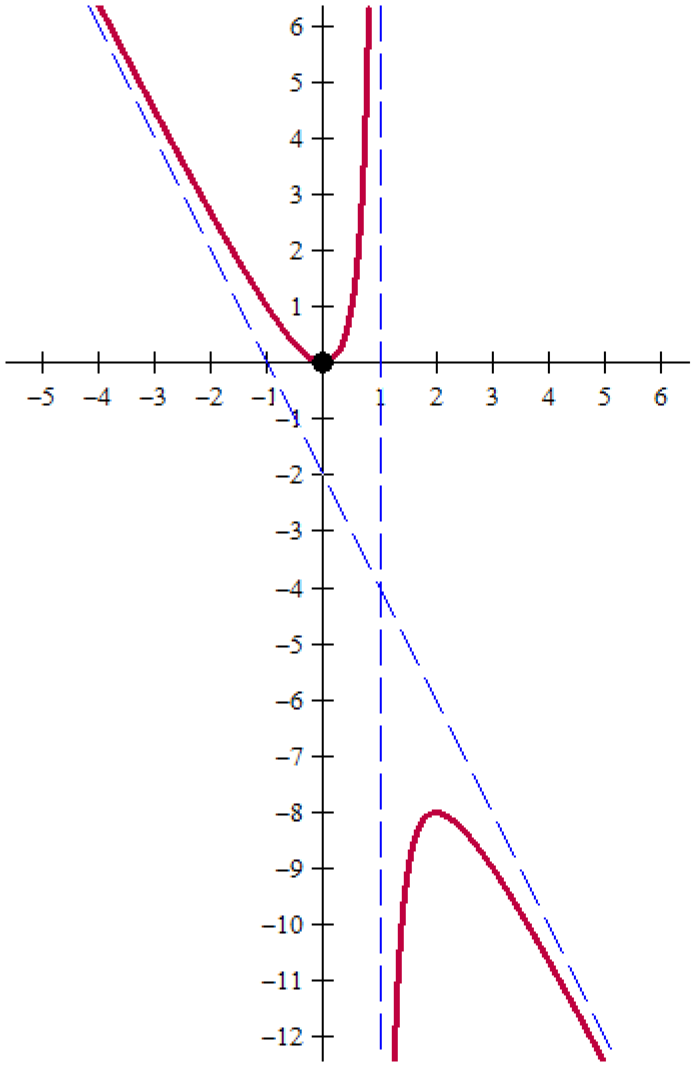
Horizontal Asymptote: None

Oblique Asymptote: yes, see next slide

Zero(s): x = 0

Holes: None

Domain: (-∞, 1)U(1, ∞)



Use long (or possibly synthetic) division to find the equation for oblique asymptotes.

$$
\begin{array}{r}
-2x \;\; - 2 \\
-x + 1 \,\overline{)\, 2x^2 + 0x + 0} \\
-(2x^2 - 2x) \quad \\
\hline
2x + 0 \\
-(-2x - 2) \\
\hline
2
\end{array}
$$

We do not care about the remainder for the equation of oblique asymptotes. The equation of the oblique asymptote is: $y = -2x - 2$

Graph the equation $f(x) = \frac{x^3-1}{2x^2-2}$ and determine its attributes.

$$f(x) = \frac{(x-1)(x^2+x+1)}{2(x+1)(x-1)} = \frac{(x^2+x+1)}{2(x+1)}$$

Vertical Asymptote: x = -1

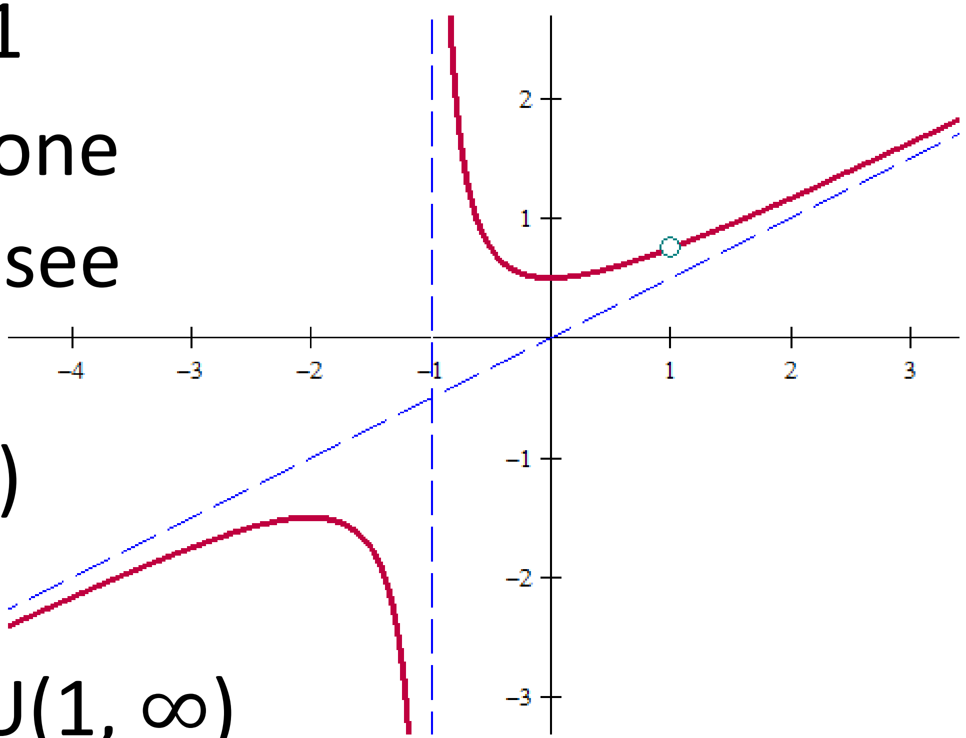
Horizontal Asymptote: None

Oblique Asymptote: yes, see next slide

Zero(s): None (imaginary)

Holes: x = 1   (1, ¾)

Domain: (-∞, -1)U(-1, 1)U(1, ∞)



Use long (or possibly synthetic) division to find the equation for oblique asymptotes.

$$
\begin{array}{r l}
 & .5x \\
2x^2 + 0x - 2 \,\big) & x^3 + 0x^2 + 0x - 1 \\
 & -(x^3 + 0x^2 - x) \\
\hline
 & x - 1
\end{array}
$$

$x - 1$ ← Remainder

We do not care about the remainder for the equation of oblique asymptotes. The equation of the oblique asymptote is: y = .5x or y = ½x