Pascal's Triangle

Pascal's Triangle is a shortcut to simplify a binomial raised to a power.   i.e. $(x + 3)^5$, $(2 - 5x)^7$, etc.

Example) Simplify $(x + 2)^5$

$(x + 2)^5 = (x + 2)^2(x + 2)^2(x + 2)$

$(x + 2)^2 = x^2 + 4x + 4$ by FOIL

$(x + 2)^5 = (x^2 + 4x + 4)(x^2 + 4x + 4)(x + 2)$

$(x + 2)^5 = (x^4 + 8x^3 + 24x^2 + 32x + 16)(x + 2)$ Distribute the trinomials

$(x + 2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$ Distribute the binomial

The make Pascal’s Triangle:

Each row has a one as its first and last term.

Any middle term is found by adding the two terms diagonally left and right above it.

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                  1
                1   1
              1   2   1
            1   3   3   1
          1   4   6   4   1
        1   5  10  10   5   1
      1   6  15  20  15   6   1
                 Etc.
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1 + 1 = 2

1 + 2 = 3

1 + 3 = 4 and 3 + 3 = 6

1 + 4 = 5 and 4 + 6 = 10

Binomial Expansion	
$(a+b)^0 =$	1
$(a+b)^1 =$	$a+b$
$(a+b)^2 =$	$a^2+2ab+b^2$
$(a+b)^3 =$	$a^3+3a^2b+3ab^2+b^3$
$(a+b)^4 =$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$
$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$	

The rows of Pascal's triangle give the coefficients of each term.

The exponents of "a" decrease (a^3, a^2, a^1, a^0)

while the exponents of "b" increase (b^0, b^1, b^2, b^3)

Expand the binomial $(3x - 4)^4$

$a = 3x$ and $b = -4$

1 4 6 4 1

Write the row for Pascal's Δ

$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

Write the formula

Substitute a and b

$1(3x)^4 + 4(3x)^3(-4) + 6(3x)^2(-4)^2 + 4(3x)(-4)^3 + 1(-4)^4$

Simplify

$81x^4 - 432x^3 + 864x^2 - 768x + 256$

Expand the binomial $(x + 2)^7$

$a = x$ and $b = 2$

1 7 21 35 35 21 7 1 — Write the row for Pascal's Δ

Write the formula

$$1a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + 1b^7$$

Substitute a and b

$$1x^7 + 7x^6(2) + 21x^5(2)^2 + 35x^4(2)^3 + 35x^3(2)^4 + 21x^2(2)^5 + 7x(2)^6 + 1(2)^7$$

Simplify

$$x^7 + 14x^6 + 84x^5 + 280x^4 + 560x^3 + 672x^2 + 448x + 128$$

Determine the coefficient of x^2 in the expansion of $(3 - x)^3$

$a = 3$ and $b = -x$

$1 \quad 3 \quad 3 \quad 1$

Write the row for Pascal's Δ

$1a^3 + 3a^2b + 3ab^2 + 1b^3$

Write the formula, and find the term with x^2

$3(3)(-x)^2$

Substitute a and b

$9x^2$

Simplify