Properties of Logarithms Expansion

If the base and argument of a log are the same, then:

$$\log_b(b) = 1$$

So, $\log_2 2 = 1,\ \log_5 5 = 1,\ \log 10 = 1,\ \ln e = 1$

The argument of a log must be positive (not negative, not zero).

What are all the possible values of x in the equation: $\log_8(x-8)$?

The argument must be positive:

$x - 8 > 0$

$x > 8$

What are all the possible values of x in the equation: $\ln(3+x^2)$?

The argument must be positive:

Since x is being squared, $3 + x^2$ is always positive.

x can be all real numbers: $(-\infty,\infty)$

If you are asked to **expand** a log, then you are supposed to use log rules to take a single log with a complicated argument and write multiple logs with simple arguments.

Multiplication rule:

$$\log_b(m * n) = \log_b m + \log_b n$$

Proof of the multiplication rule:

Given: $x^m = A,$ then $\log_x(A) = m$

Given: $x^n = B,$ then $\log_x(B) = n$

Given: $x^p = A * B,$ then $\log_x(A * B) = p$

Because $x^p = A * B$, we also have $x^p = x^m * x^n$ by substitution.

By exponent laws: $x^p = x^m * x^n \rightarrow x^p = x^{m+n}$

Because $x^p = x^{m+n}$ has equal bases, it also has equal exponents: p = m + n.

By substitution: $\log_x(A * B) = \log_x(A) + \log_x(B)$

Expand:

$\log_2(4x)$

$\log_2(4) + \log_2(x)$

$\ln(3xy)$

$\ln(3) + \ln(x) + \ln(y)$

$\log_5(23x)$

$\log_5(23) + \log_5(x)$

Expand:

$\log_6(5^3)$ $\log_6(5 \cdot 5 \cdot 5)$

$\log_6(5) + \log_6(5) + \log_6(5)$

$3\log_6(5)$

$\log(x^4)$ $\log(x \cdot x \cdot x \cdot x)$

$\log(x) + \log(x) + \log(x) + \log(x)$

$4\log(x)$

Exponent rule:

$$\log_b(m^p) = p \log_b m$$

Expand:

$\log_2(y^{12})$

$12\log_2(y)$

$\ln(3x^5)$

$\ln(3) + \ln(x^5)$

$\ln(3) + 5\ln(x)$

$\log_4(7x^2y^3z)$

$\log_4(7) + \log_4(x^2) + \log_4(y^3) + \log_4(z)$

$\log_4(7) + 2\log_4(x) + 3\log_4(y) + \log_4(z)$

Expand:

$$\log_3\left(\frac{x}{y}\right)$$

$$\log_3(xy^{-1})$$

$$\log_3(x) + \log_3(y^{-1})$$

$$\log_3(x) - \log_3(y)$$

$$\log_5\left(\frac{1}{\sqrt{x} * y^2}\right)$$

$$\log_5(x^{-1/2} * y^{-2})$$

$$\log_5\left(x^{-1/2}\right) + \log_5(y^{-2})$$

$$-1/2\log_5(x) - 2\log_5(y)$$

Expand:

$$\log_3\left(\frac{3x^4}{z}\right)^3$$

$$\log(3x^4z^{-1})^3$$

$$\log(3^3x^{12}z^{-3})$$

$$\log(3^3)+\log(x^{12})+\log(z^{-3})$$

$$\log(27)+12\log(x)-3\log(z)$$

Expand:

$\log_5(5 + x)$

It's a trap! You cannot expand addition or subtraction in the argument.