PSAT Warm-Up

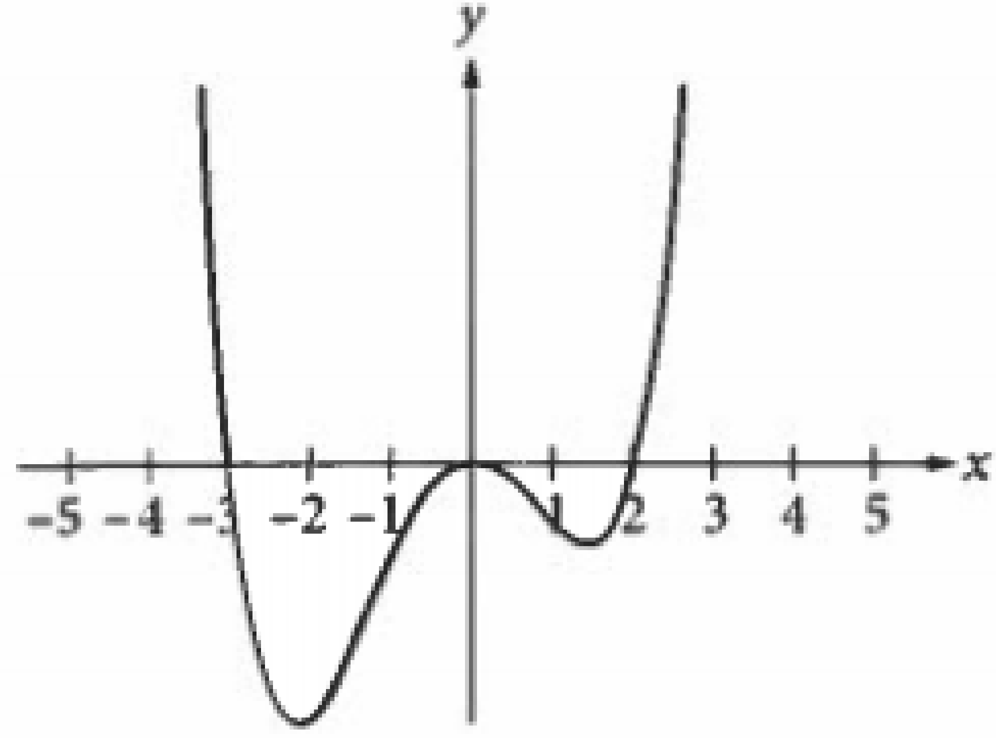
Which of the following could be an equation for the graph shown in the xy-plane below? (Non-calc)

(A) $y = x(x + 3)(x - 2)$

(B) $y = x(x + 2)(x - 3)$

(C) $y = x^2(x + 3)(x - 2)$

(D) $y = x^2(x + 2)(x - 3)$



Quadratic Formula

When solving a quadratic:

Try Factoring

Completing the square

Quadratic Formula

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The $b^2 - 4ac$ part of the Quadratic Formula is called the **discriminant**.

The discriminant does not include the square root.

The discriminant tells how many and what kinds of solutions a quadratic equation will have.

How many and what type of solutions will the equation $y = 4x^2 - 3x - 12$ have?

$b^2 - 4ac$	Use the discriminant
$(-3)^2 - 4(4)(-12)$	Sub. the values of a, b, c
$9 + 192$	Square and multiply
201	Combine like terms

Because the discriminant is positive, there are two real solutions.

How many and what type of solutions will the equation $y = 2x^2 + 8x + 8$ have?

$b^2 – 4ac$	Use the discriminant
$(8)^2 – 4(2)(8)$	Sub. the values of a, b, c
$64 – 64$	Square and multiply
0	Combine like terms

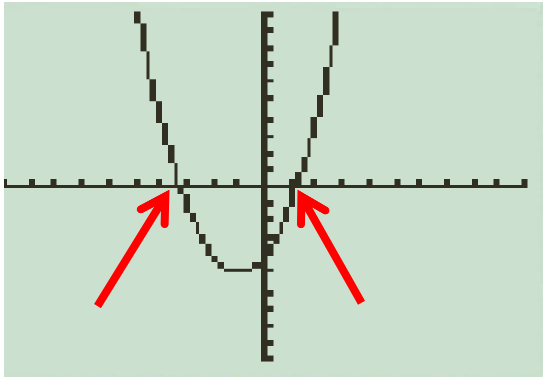
Because the discriminant is zero, there is one real solution (a double root).

How many and what type of solutions will the equation $y = 3x^2 - 6x + 5$ have?

$b^2 - 4ac$	Use the discriminant
$(-6)^2 - 4(3)(5)$	Sub. the values of a, b, c
$36 - 60$	Square and multiply
-24	Combine like terms

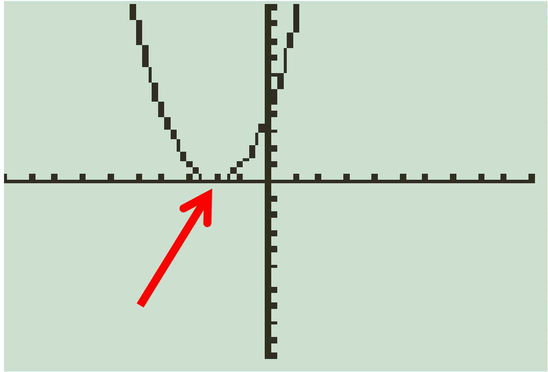
Because the discriminant is negative, there are two non-real solutions.

If the discriminant is positive, there will be two real solutions.



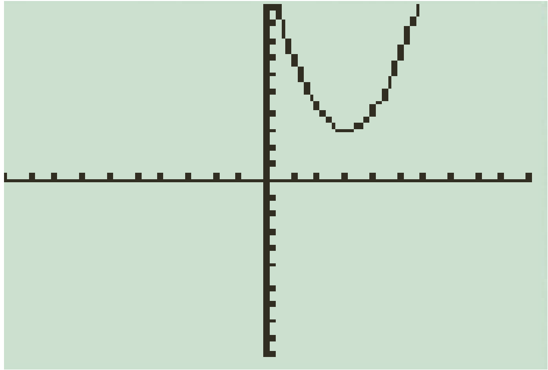
Crosses the x-axis two different places

If the discriminant = 0, then there is one unique solution called a double root (it counts twice).



The vertex sits on the x-axis

If the discriminant is negative, then there are two distinct non-real solutions.



Never crosses the x-axis

Non-real numbers include imaginary numbers and complex numbers.

Imaginary numbers come from the square root of negative numbers. Imaginary numbers are notated using the letter i.

Example) $\sqrt{-16} = \ 4i \quad and \ -4i$

A **complex number** has both an imaginary and real part.

Example) $4 - 2i$ and $18 + 5.5i$ are both complex

Contrary to their name, imaginary numbers are not "made-up" numbers.

Imaginary numbers are named "imaginary" because they are the opposite of real numbers.

Real numbers: $\sqrt{20}$ Imaginary numbers: $\sqrt{-20}$

Imaginary numbers are very real. They just have an unfortunate name.

"What's in a name? that which we call a rose
By any other name would smell as sweet"

"What's in a name? that which we call an imaginary number
By any other name would still be a number"

Why imaginary numbers?

Imaginary numbers have many applications in (much) higher levels of mathematics.

Engineers use imaginary numbers to solve equations that relate functions with their rates of growth.

Physicists use imaginary numbers when calculating quantum mechanics.

Imaginary numbers sometimes make solving difficult equations easier.

Imaginary numbers let Algebra 2 students find all of the roots for x^2, x^3, x^4, etc. equations.

Find the zeros of $x^2 + 3x = 7$ using the Quadratic Formula.

$1x^2 + 3x - 7 = 0$ — Set f(x) = 0.

$b^2 - 4ac$ — Evaluate the discriminant

$(3)^2 - 4(1)(-7)$ — Sub. the values of a, b, c

$9 + 28$ — Square and multiply

37 — Combine like terms

$$x = \frac{-3 \pm \sqrt{37}}{2}$$

Sub into the quadratic formula.

$$x = \frac{-3 + \sqrt{37}}{2} \qquad x = \frac{-3 - \sqrt{37}}{2}$$

Write as two answers: one plus, one minus

$$x \approx 1.541 \qquad x \approx -4.541$$

Simplify to a decimal (not necessary)

Find the zeros of $4x^2 + 3x + 2$ using the Quadratic Formula.

$4x^2 + 3x + 2 = 0$ — Set f(x) = 0.

$b^2 - 4ac$ — Evaluate the discriminant

$(3)^2 - 4(4)(2)$ — Sub. the values of a, b, c

$9 - 32$ — Square and multiply

-23 — Combine like terms

$$x = \frac{-3 + \sqrt{-23}}{2(4)}$$

Sub into the quadratic formula.

$$x = \frac{-3 + \sqrt{-23}}{8} \qquad x = \frac{-3 - \sqrt{-23}}{8}$$

Write as two answers: one plus, one minus

$$x = \frac{-3}{8} + \frac{\sqrt{23}}{8}i \qquad x = \frac{-3}{8} - \frac{\sqrt{23}}{8}i$$

Write as two fractions, change -23 to +23 by putting i.

$$x \approx -.375 + .599i \qquad x \approx -.375 - .599i$$

Simplify to a decimal (not necessary)

Find the zeros of $x^2 + 40 = 12x$ using the Quadratic Formula.

$x^2 - 12x + 40 = 0$ — Set f(x) = 0.

$b^2 - 4ac$ — Evaluate the discriminant

$(-12)^2 - 4(1)(40)$ — Sub. the values of a, b, c

$144 - 160$ — Square and multiply

-16 — Combine like terms

$$x = \frac{12 + \sqrt{-16}}{2(1)}$$

Sub into the quadratic formula.

$$x = \frac{12 + \sqrt{-16}}{2} \qquad x = \frac{12 - \sqrt{-16}}{2}$$

Write as two answers: one plus, one minus

$$x = \frac{12}{2} + \frac{\sqrt{16}}{2}i \qquad x = \frac{12}{2} - \frac{\sqrt{16}}{2}i$$

Write as two fractions, change -23 to +23 by putting i.

$$x = 6 + 2i \qquad x = 6 - 2i$$

Simplify to a decimal (not necessary)