# Reasonable Domain and Range

Sometimes you should restrict the domain and range of a function based on the context of the problem because it wouldn't make sense to use the entire domain/range.

Word problems that deal with time typically have their domain restricted so that $t \geq 0$.

Problems with height often restrict their range so that $h \geq 0$.

**Ex 1)** An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where $s$ is in meters and t is in seconds. What would be a reasonable domain and range for the object flight?

$$-4.9t^2 + 19.6t + 58.8 = 0$$

$$x = \frac{-19.6 \pm \sqrt{(19.6)^2 - 4(-4.9)(58.8)}}{2(-4.9)}$$

Use the quadratic formula to find the zeros (i.e. when the height is zero).

$$x = \frac{-19.6 + \sqrt{1536.64}}{-9.8} \qquad x = \frac{-19.6 - \sqrt{1536.64}}{-9.8}$$

$$x = -2 \qquad x = 6$$

The object starts its flight at time t = 0 and will stop its flight when it hits the ground.

Domain: [0, 6]

The highest point of the flight will occur at the vertex.

$$x = \frac{-b}{2a}$$

$$x = \frac{-(19.6)}{2(-4.9)}$$

$$x = \frac{-19.6}{-9.8} = 2$$

$$s(2) = -4.9(2)^2 + 19.6(2) + 58.8$$

$$s(2) = 78.4$$

Vertex: (2, 78.4)

The object reaches a maximum of 78.4 and will land on the ground at a height of 0.
Range: [0, 78.4]

**Ex 2)** An object in launched directly upward at 64 feet per second (ft/s) from a platform 80 feet high. The equation for its path is $h(t) = -16t^2 + 64t + 80$. What timeframe will the object's height be above 120 feet?

$$-16t^2 + 64t + 80 = 120$$

$$-16t^2 + 64t - 40 = 0$$

Use the quadratic formula to find when the height is 120.

$$x = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(-40)}}{2(-16)}$$

$$x = \frac{-64 + \sqrt{1536}}{-32} \qquad x = \frac{-64 - \sqrt{1536}}{-32}$$

$$x = .775 \qquad x = 3.225$$

The object has a height of 120 at x = .775 and x = 3.225. The object is above 120 between those two times because the parabola faces downward. Use parenthesis because it is strictly above 120.
Domain: (0.775, 3.225)

**Ex 3)** An object is launched from ground level directly upward at 39.2 m/s. Its path is modeled by the equation $h(t) = -4.9t^2 + 39.2t$. Determine a range of heights for the object between 3 and 6 seconds.

Find the height at t = 2 and t = 5

$$h(3) = -4.9(3)^2 + 39.2(3)$$

$$h(3) = 73.5$$

$$h(6) = -4.9(6)^2 + 39.2(6)$$

$$h(6) = 58.8$$

Consider the vertex because it is between 3 and 6

$$x = \frac{-b}{2a} \qquad x = \frac{-(39.2)}{2(-4.9)}$$

$$x = \frac{-39.2}{-9.8} = 4$$

$$h(4) = -4.9(4)^2 + 39.2(4)$$

$$h(4) = 78.4$$

Vertex: (4, 78.4)

The object's lowest height is 58.8 and its highest height is 78.4 (at the vertex). Use a bracket on 78.4 because the object does reach this height.
Range: (58.8, 78.4]