

## Matrix Review

Make sure you can do all of the following and understand the rules for multiplying, adding, subtracting, etc. for the always, sometimes, and never type questions.

$$P = \begin{bmatrix} 3 & -5 & 2 \\ -4 & 1 & 3 \end{bmatrix}_{2 \times 3} \quad Q = \begin{bmatrix} 2 & 3x \\ 4x & 5 \end{bmatrix}_{2 \times 2} \quad R = \begin{bmatrix} 6 & -8 & 4x \\ -10 & 2x^2 & 4 \end{bmatrix}_{2 \times 3}$$

Use the matrices above to evaluate. If not possible, explain why.

1)  $P - 2Q$

Not possible, the dimensions do not match.

$$2 \times 3 - 2 \times 2$$

3)  $\frac{1}{2}R - \frac{1}{3}P$

$$\begin{bmatrix} 3 & -4 & 2x \\ -5 & x^2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & \frac{5}{3} & -\frac{2}{3} \\ \frac{4}{3} & \frac{1}{3} & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2\bar{3} & 2x - \frac{2}{3} \\ -3\bar{6} & x^2 - \frac{1}{3} & 1 \end{bmatrix}$$

2)  $QR$

$$\begin{bmatrix} 2 & 3x \\ 4x & 5 \end{bmatrix} \begin{bmatrix} 6 & -8 & 4x \\ -10 & 2x^2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2(6) + 3x(-10) & 2(-8) + 3x(2x^2) & 2(4x) + 3x(4) \\ 4x(6) + 5(-10) & 4x(-8) + 5(2x^2) & 4x(4x) + 5(4) \end{bmatrix}$$

$$\begin{bmatrix} -30x + 12 & 6x^3 - 16 & 20x \\ 24x - 50 & 10x^2 - 32x & 16x + 20 \end{bmatrix}$$

4)  $\frac{1}{2}(2P + R) \rightarrow P + \frac{1}{2}R$

$$\begin{bmatrix} 3 & -5 & 2 \\ -4 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -4 & 2x \\ -5 & x^2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -9 & 2x + 2 \\ -9 & x^2 + 1 & 5 \end{bmatrix}$$

Find the inverse for problems 5 and 6.

5)  $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \det(A) = 1 - (-1) = 2$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

6)  $\begin{bmatrix} -\frac{1}{4} & 3 \\ -\frac{2}{3} & 6 \end{bmatrix} \det(A) = -1.5 - (-2) = .5$

$$A^{-1} = \frac{1}{.5} \begin{bmatrix} 6 & -3 \\ \frac{2}{3} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ \frac{4}{3} & -\frac{1}{2} \end{bmatrix}$$

7) A pet stroller comes in two sizes. Two stores have inventories as shown in the first table. Find the total cost of the pet strollers for each store.

Pet Stroller Inventory		
	Standard	Large
Store 1	7	6
Store 2	9	13

Pet Stroller Profits			
	Revenue (\$)	Store Cost (\$)	Profit (\$)
Standard	125	85	40
Large	175	110	65

Multiply the two matrices together. Identify what entries  $a_{12}$  and  $a_{23}$  mean in the context of the problem.

$$A = \begin{bmatrix} 1925 & 1255 & 670 \\ 3400 & 2195 & 1205 \end{bmatrix}$$

$a_{12} = \$1255$  store 1's total store cost

$a_{23} = \$1205$  store 2's total profit

Evaluate using the matrices below for problems 8-11. If not possible, explain why.

$$E = \begin{bmatrix} 1 & -2 & -1 \\ 5 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}_{3 \times 3} \quad F = [0.5 \quad 0.75 \quad -1]_{1 \times 3} \quad G = \begin{bmatrix} 0 & 2x \\ 2x & -1 \end{bmatrix}_{2 \times 2} \quad H = \begin{bmatrix} -1 & 4 \\ 2 & 0 \\ 0 & -1 \end{bmatrix}_{3 \times 2}$$

8) EF  
Not possible, inside dimensions  
do not match  
 $3 \times 3$   $1 \times 3$

9) FH  $\begin{bmatrix} .5 & .75 & -1 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 2 & 0 \\ 0 & -1 \end{bmatrix}$

$$[.5(-1) + .75(2) + -1(0) \quad .5(4) + .75(0) + -1(-1)]$$

$$\boxed{[1 \quad 3]}$$

10) HG  $\begin{bmatrix} -1 & 4 \\ 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 2x \\ 2x & -1 \end{bmatrix} = \begin{bmatrix} -1(0) + 4(2x) & -1(2x) + 4(-1) \\ 2(0) + 0(2x) & 2(2x) + 0(-1) \\ 0(0) + -1(2x) & 0(2x) + -1(-1) \end{bmatrix}$

$$\boxed{\begin{bmatrix} 8x & -2x-4 \\ 0 & 4x \\ -2x & 1 \end{bmatrix}}$$

11)  $G^{-1}$   
 $\det(G) = 0 - (4x^2) = -4x^2$   
 $G^{-1} = \frac{1}{-4x^2} \begin{bmatrix} -1 & -2x \\ -2x & 0 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{1}{4x^2} & \frac{1}{2x^2} \\ \frac{1}{2x^2} & 0 \end{bmatrix}}$

12) Find D =  $\begin{bmatrix} 4 & -2 & 1 & 4 & -2 \\ 3 & 2 & 1 & 3 & 2 \\ -1 & 1 & 3 & -1 & 1 \end{bmatrix}$

$$(4(2)(3) + -2(1)(-1) + 1(3)(1)) - (-1(2)(1) + 1(1)(4) + 3(3)(-2))$$

$$(24 + 2 + 3) - (-2 + 4 - 18)$$

$$29 - (-16)$$

$$\boxed{45}$$

13) Multiply  $\begin{bmatrix} 1 & x \\ 2x & -x \end{bmatrix} * \begin{bmatrix} 3x & 2 \\ 0 & 2x \end{bmatrix}$

$$\begin{bmatrix} 1(3x) + x(0) & 1(2) + x(2x) \\ 2x(3x) + -x(0) & 2x(2) + -x(2x) \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 3x & 2x^2 + 2 \\ 6x^2 & -2x^2 + 4x \end{bmatrix}}$$

Write and solve a matrix equation for the system.

14)  $\begin{cases} \frac{3}{2}x = 20 + y \rightarrow \frac{3}{2}x - y = 20 \\ x + 6y = 80 \end{cases}$

$$\begin{bmatrix} 1.5 & -1 \\ 1 & 6 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 80 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ 1 & 6 \end{bmatrix}^{-1} * \begin{bmatrix} 20 \\ 80 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \end{bmatrix} \quad \begin{matrix} x=20 \\ y=10 \end{matrix}$$

15) Find the Determinant of  $\begin{bmatrix} 6x^2 & -6x+2x^2 \\ 3x & x-3 \end{bmatrix}$

$$6x^2(x-3) - (3x(-6x+2x^2))$$

$$(6x^3 - 18x^2) - (-18x^2 + 6x^3)$$

$$6x^3 - 18x^2 + 18x^2 - 6x^3$$

$$\boxed{0}$$

16) Find the value of x so that the matrix does not have an inverse:

$$\begin{bmatrix} 7 & x \\ 3 & 6 \end{bmatrix} \det A = 7(6) - 3(x)$$

$$\det(A) = 42 - 3x$$

$$42 - 3x = 0$$

$$-3x = -42$$

$$\boxed{x = 14}$$

17) Solve the following system using Gauss Elimination. Use your calculator to check that your answer is correct.

$$\begin{cases} x + 3y - 3z = 12 \\ 3x - y + 4z = 0 \\ -x + 2y - z = 1 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & -3 & 12 \\ 3 & -1 & 4 & 0 \\ -1 & 2 & -1 & 1 \end{array} \right]$$

$$\boxed{(3, 1, -2)}$$

$$\begin{aligned} 5z &= -10 \\ z &= -2 \end{aligned}$$

$$\begin{array}{r} -3R_1 + R_2 \rightarrow R_2 \\ \begin{array}{cccc} -3 & -9 & 9 & -36 \\ + 3 & -1 & 4 & 0 \\ \hline 0 & -10 & 13 & -36 \end{array} \end{array}$$

$$\begin{array}{r} R_1 + R_3 \rightarrow R_3 \\ \begin{array}{cccc} 1 & 3 & -3 & 12 \\ + -1 & 2 & -1 & 1 \\ \hline 0 & 5 & -4 & 13 \end{array} \end{array}$$

$$\begin{array}{r} R_2 + 2R_3 \rightarrow R_3 \\ \begin{array}{cccc} 0 & -10 & 13 & -36 \\ + 0 & 10 & -8 & 26 \\ \hline 0 & 0 & 5 & -10 \end{array} \end{array}$$

$$\begin{aligned} -10y + 13z &= -36 \\ -10y + 13(-2) &= -36 \\ -10y - 26 &= -36 \\ -10y &= -10 \\ y &= 1 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & -3 & 12 \\ 0 & -10 & 13 & -36 \\ -1 & 2 & -1 & 1 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & -3 & 12 \\ 0 & -10 & 13 & -36 \\ 0 & 5 & -4 & 13 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & -3 & 12 \\ 0 & -10 & 13 & -36 \\ 0 & 0 & 5 & -10 \end{array} \right]$$

$$\begin{aligned} x + 3y - 3z &= 12 \\ x + 3(1) - 3(-2) &= 12 \\ x + 3 + 6 &= 12 \\ x + 9 &= 12 \\ x &= 3 \end{aligned}$$

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$$\text{rref} \left[ \begin{array}{ccc|c} 1 & 3 & -3 & 12 \\ 3 & -1 & 4 & 0 \\ -1 & 2 & -1 & 1 \end{array} \right] \rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$x = 3 \quad y = 1 \quad z = -2$$