REVIEW Polynomials

Name: Answer Key

1. Find the first 8 rows of Pascal's Triangle and use it to expand $(2a+5)^6$. $a=2a \quad b=5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

$1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$

$(2a)^6 + 6(2a)^5(5) + 15(2a)^4(5)^2 + 20(2a)^3(5)^3 + 15(2a)^2(5)^4 + 6(2a)(5)^5 + (5)^6$

$\boxed{64a^6 + 960a^5 + 6000a^4 + 20000a^3 + 37500a^2 + 37500a + 15625}$

Find the specified term of each expansion

2) $(a+2)^4$; 3rd term $a=a \quad b=2$

$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$6(a)^2(2)^2$

$\boxed{24a^2}$

3) $(3x+4)^5$; 5th term $a=3x \quad b=4$

$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

$5(3x)(4)^4$

$\boxed{3840x}$

Find each product.

4) $(3c^2-6c+2)(4c^2-6c+14)$

$12c^4 - 18c^3 + 42c^2 - 24c^3 + 36c^2 - 84c + 8c^2 - 12c + 28$

$\boxed{12c^4 - 42c^3 + 86c^2 - 96c + 28}$

5) $(a-3)(2-5a+a^2)$

$2a - 5a^2 + a^3 - 6 + 15a - 3a^2$

$\boxed{a^3 - 8a^2 + 17a - 6}$

6) Mr. Silva manages a manufacturing plant. From 1990 through 2005 the number of units produced (in thousands) can be modeled by $N(x) = 0.02x^2 + 0.2x + 3$. The average cost per unit (in dollars) can be modeled by $C(x) = -0.004x^2 - 0.1x + 3$. Write a polynomial $T(x)$ that can be used to model the total costs and evaluate T(5.2).

$T(x) = N(x) \cdot C(x)$

$T(x) = (.02x^2 + .2x + 3)(-.004x^2 - .1x + 3)$

$T(x) = -.00008x^4 - .002x^3 + .06x^2 - .0008x^3 - .02x^2 + .6x - .012x^2 - .3x + 9$

$\boxed{T(x) = -.00008x^4 - .0028x^3 + .028x^2 + .3x + 9}$ $\boxed{T(5.2) = 10.865 = \$10,865}$

Use long division to find the quotient and the remainder.

7) $(-y^2 + 4y^3 + 25) \div (2y^2 - 3)$

$$\begin{array}{r} 2y - \frac{1}{2} \\ 2y^2+0y-3 \overline{\smash{)}\, 4y^3 - y^2 + 0y + 25} \\ -(4y^3 + 0y - 6y) \\ \hline -y^2 + 6y + 25 \\ -(-y^2 + 0y + 1.5) \\ \hline 6y + 23.5 \end{array}$$

$$\boxed{2y - \frac{1}{2} + \frac{6y+23.5}{2y^2-3}}$$

8) $(15x^4 + 8x - 12) \div (3x^2 + 1)$

$$\begin{array}{r} 5x^2 \quad - 5/3 \\ 3x^2+0x+1 \overline{\smash{)}\, 15x^4 + 0x^3 + 0x^2 + 8x - 12} \\ -(15x^4 + 0x^3 + 5x^2) \\ \hline -5x^2 + 8x - 12 \\ -(-5x^2 + 0x - 5/3) \\ \hline 8x - 10.\overline{3} \end{array}$$

$$\boxed{5x^2 - \frac{5}{3} + \frac{8x - 10.\overline{3}}{3x^2+1}}$$

9) Write an expression that represents the area of the top face of a cylinder when the height is $x + 2$ and the volume of the cylinder is $x^3 - x^2 - 6x$. $A = \dfrac{V}{h}$.

$$\begin{array}{r} x^2 - 3x \\ x+2 \overline{\smash{)}\, x^3 - x^2 - 6x + 0} \\ -(x^3 + 2x^2) \\ \hline -3x^2 - 6x \\ -(-3x^2 - 6x) \\ \hline 0x + 0 \end{array}$$

$$\boxed{A = x^2 - 3x}$$

Factor the following polynomials.

10) $2x^3 + x^2 + 8x + 4$

$x^2(2x+1) + 4(2x+1)$

$\boxed{(x^2+4)(2x+1)}$

11) $64x^3 - 8$

$8(8x^3 - 1)$

$a = 2x \quad b = 1$

$(a-b)(a^2+ab+b^2)$

$\boxed{8(2x-1)(4x^2+2x+1)}$

12) $54x^3 + 250y^6$

$2(27x^3 + 125y^6)$

$a = 3x \quad b = 5y^2$

$(a+b)(a^2-ab+b^2)$

$\boxed{2(3x+5y^2)(9x^2-15xy^2+25y^4)}$

13) Write the simplest polynomial with roots –1, 1/2 multiplicity of 3, and 4. What is the degree of the polynomial?

$P(x) = (x+1)(x-\frac{1}{2})^3(x-4)$ degree: 5

14) Write the simplest polynomial with a double root at x = 2 and a triple root at x = 3. What is the degree of the polynomial?

$P(x) = (x-2)^2(x-3)^3$ degree: 5

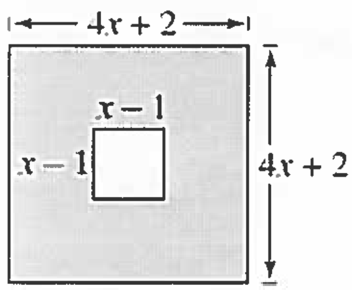
15) Describe the end behavior of an even function whose leading coefficient is negative.

Both f(x) values will go to negative infinity.

16) Describe the end behavior of an odd function whose leading coefficient is positive.

As $x \to \infty$, $f(x) \to \infty$ and As $x \to -\infty$, $f(x) \to -\infty$

17) Find the Area of the shaded region.



$(4x+2)(4x+2)$

$16x^2 + 8x + 8x + 4$

$16x^2 + 16x + 4$

$(x-1)(x-1)$

$x^2 - x - x + 1$

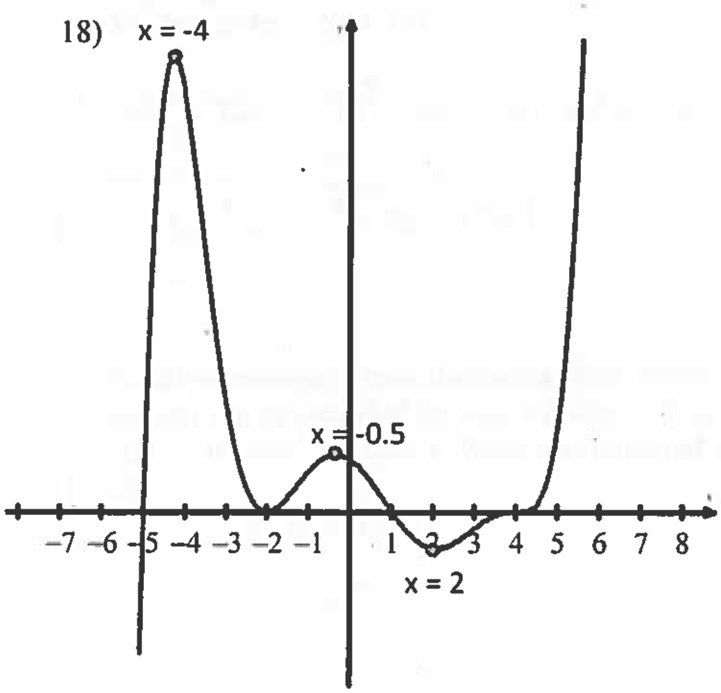
$x^2 - 2x + 1$

$(16x^2 + 16x + 4) - (x^2 - 2x + 1)$

$16x^2 + 16x + 4 - x^2 + 2x - 1$

$\boxed{15x^2 + 18x + 3}$

18)



Approximate Roots with multiplicity: X = -5 mult. of 1, X = -2 mult. of 2, X = 1 mult. of 1, X = 4 mult. of 3

Increasing: $(-\infty, -4) \cup (-2, -.5) \cup (2, 4) \cup (4, \infty)$

Decreasing: $(-4, -2) \cup (-.5, 2)$

Relative Maximum(s): X = -4 and X = -.5

Relative Minimum(s): X = -2 and X = 2

End behavior: As $x \to \infty$, $f(x) \to \infty$ As $x \to -\infty$, $f(x) \to -\infty$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Odd or even function: odd Degree: 7

Sign of leading coefficient: positive

Equation of the function written in factored form:

$P(x) = (x+5)(x+2)^2(x-1)(x-4)^3$