

Rational Functions Review

Name Answer Key

1) The mass m in kilograms of a bronze statue varies directly as its volume V in cubic centimeters. If a statue made from 1000 cm^3 of bronze has a mass of 8.7 kg , what is the mass of a statue made from 4500 cm^3 of bronze?

$$m = kV$$

$$8.7 = k(1000)$$

$$k = .0087$$

$$m = .0087V$$

$$m = .0087(4500)$$

$$m = 39.15 \text{ kg}$$

2) The time t in hours needed to clean the rides at an amusement park varies inversely with the number of workers n . If 6 workers can clean the rides in 6 hours, how many hours will it take 10 workers to clean the rides?

$$t = \frac{k}{n}$$

$$6 = \frac{k}{6}$$

$$k = 36$$

$$t = \frac{36}{n}$$

$$t = 3.6 \text{ hours}$$

3) Determine whether the data set represents a direct variation or inverse variation, determine the constant coefficient and the equation for this set.

x	2	5	10
y	25	10	5

One variable increases while the other decreases. Likely inverse variation.

$$y = \frac{k}{x}$$

$$25 = \frac{k}{2}$$

$$k = 50$$

$$y = \frac{50}{x}$$

$$10 = \frac{50}{5} \checkmark$$

$$5 = \frac{50}{10} \checkmark$$

Identify any places of discontinuity for each rational function (i.e. holes and vertical asymptotes).

4) $f(x) = \frac{2x^3 - 18x}{x^2 + 3x - 18} = \frac{2x(x^2 - 9)}{(x-3)(x+6)}$

Hole: $x = 3$
VA: $x = -6$

$$\frac{2x(x+3)(x-3)}{(x+6)(x-3)}$$

5) $f(x) = \frac{x^2 + x - 12}{x^2 - 16} = \frac{(x+4)(x-3)}{(x+4)(x-4)}$

Hole: $x = -4$
VA: $x = 4$

Identify all asymptotes (vertical, horizontal, oblique) of the following functions.

6) $f(x) = \frac{(x^2 - 3x + 2)(x - 3)}{(x - 1)(x^2 - 5x + 6)}$

$$f(x) = \frac{(x-2)(x-1)(x-3)}{(x-1)(x-2)(x-3)}$$

$$f(x) = 1$$

VA: None
HA: $y = 1$
Oblique: None

7) $f(x) = \frac{x^2 - 2x - 3}{3x^2 - 3x}$

$$f(x) = \frac{(x-3)(x+1)}{3x(x-1)}$$

VA: $x = 0, x = 1$
HA: $y = \frac{1}{3}$
Oblique: None

8) $f(x) = \frac{-3x^2 + 2}{x - 1}$

$$x-1 \overline{) \begin{array}{r} -3x^2 + 0x + 2 \\ -(-3x^2 + 3x) \\ \hline -3x + 2 \\ -(-3x + 3) \\ \hline -1 \end{array}}$$

-1 ← Remainder

VA: $x = 1$
HA: None
Oblique: $y = -3x - 3$

where numerator equals zero

Find the zeros of the following functions.

9) $f(x) = \frac{3x+2}{x-2}$

$3x+2=0$

$3x=-2$

$x = -2/3$

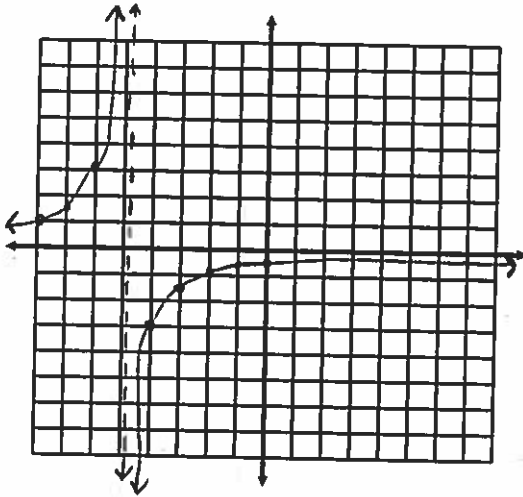
10) $f(x) = \frac{x^2-x}{x^2-25} = \frac{x(x-1)}{(x+5)(x-5)}$

$x=0$ and $x-1=0$

$x=0$ and $x=1$

Identify the following attributes of each function. Then graph.

11) $g(x) = \frac{-3}{x+5}$



Vertical asymptote(s): $x = -5$

Horizontal asymptote(s): $y = 0$

Domain: $(-\infty, -5) \cup (-5, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Holes: None

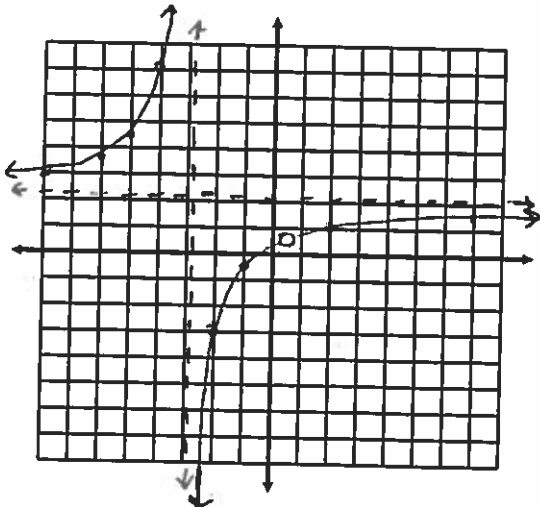
Zeros: None

Y-Int: $(0, -3/5)$

12) $f(x) = \frac{4x^2-1}{2x^2+5x-3} = \frac{(2x+1)(2x-1)}{(x+3)(2x-1)} = \frac{2x+1}{x+3}$

hole at $x = 1/2$

$x^2 + 5x - 6$
 $(x-1)(x+6)$
 $(2x-1)(x+3)$



Vertical asymptote(s): $x = -3$

Horizontal asymptote(s): $y = 2$

Domain: $(-\infty, -3) \cup (-3, 1/2) \cup (1/2, \infty)$

Range: $(-\infty, .571) \cup (.571, 2) \cup (2, \infty)$

Holes: $(1/2, .571)$

Zeros: $(-1/2, 0)$

Y-Int: $(0, 1/3)$

Write the equation of a rational function with the given characteristics:

13) zero at 0, vertical asymptotes at $x = -3$ and $x = 3$, and holes at $x = -1$ and $x = 1$

$$f(x) = \frac{x(x+1)(x-1)}{(x+3)(x-3)(x+1)(x-1)}$$

14) zeros at $x=3$, vertical asymptotes at $x=7$ and $x=2$, and horizontal asymptotes at $y=1$

$$g(x) = \frac{(x-3)^2}{(x-7)(x-2)}$$

squared so numerator and denominator have the same degree

Solve. Identify any extraneous solutions.

15) $\frac{x^2+3x-10}{x-2} = 7$

$$\frac{(x+5)(x-2)}{(x-2)} = 7 \quad \text{hole at } x=2$$

$$x+5 = 7$$

$$x = 2 \leftarrow \text{extraneous}$$

No solution

16) $\frac{3}{x^2-9} + \frac{5}{x+3} = \frac{10}{x+3}$ LCD: $(x+3)(x-3)$

$$\frac{3}{(x+3)(x-3)} + \frac{5}{x+3} = \frac{10}{x+3}$$

$$\frac{3}{(x+3)(x-3)}(x+3)(x-3) + \frac{5}{x+3}(x+3)(x-3) = \frac{10}{x+3}(x+3)(x-3)$$

$$3 + 5(x-3) = 10(x-3)$$

$$3 + 5x - 15 = 10x - 30$$

$$18 = 5x$$

$$x = 3.6$$

Solve each rational inequality. Write the solution using interval notation.

17) $\frac{x+4}{x} > -2$ LCD: x

$$\frac{x+4}{x}(x) = -2(x)$$

$$x+4 = -2x$$

$$4 = -3x$$

$$x = -\frac{4}{3} \quad \text{VA: } x=0$$

x	-2	$-\frac{4}{3}$	$-\frac{1}{2}$	0	1
f(x)	True	zero	False	DNE	True

$$(-\infty, -\frac{4}{3}) \cup (0, \infty)$$

18) $\frac{2}{x-3} < 4$ LCD: $x-3$

$$\frac{2}{x-3}(x-3) = 4(x-3)$$

$$2 = 4x - 12$$

$$14 = 4x$$

$$x = 3.5 \quad \text{VA: } x=3$$

x	0	3	3.25	3.5	5
f(x)	True	DNE	False	zero	True

$$(-\infty, 3) \cup (3.5, \infty)$$

Multiply or divide. Assume that all expressions are defined.

$$19) \frac{x+3}{x+2} \cdot \frac{2x-4}{x^2-9}$$

$$\frac{(x+3)}{(x+2)} \cdot \frac{2(x-2)}{(x+3)(x-3)}$$

$$\boxed{\frac{2(x-2)}{(x+2)(x-3)}}$$

$$20) \frac{3x^2+6x-24}{x^2-x-20} \div \frac{3x^3-9x^2+6x}{x}$$

$$\frac{3(x+4)(x-2)}{(x-5)(x+4)} \cdot \frac{x}{3x(x-2)(x-1)}$$

$$\boxed{\frac{1}{(x-5)(x-1)}}$$

Add or subtract. Identify any x -values for which the expression is undefined.

$$21) \frac{5x-1}{x+3} + \frac{3x}{2x+6} \quad \text{LCD: } 2(x+3)$$

$$\frac{5x-1}{(x+3)} + \frac{3x}{2(x+3)}$$

$$\frac{2}{2} \cdot \frac{(5x-1)}{(x+3)} + \frac{3x}{2(x+3)}$$

$$\frac{10x-2}{2(x+3)} + \frac{3x}{2(x+3)}$$

$$\boxed{\frac{13x-2}{2(x+3)}}$$

$$22) \frac{3x}{x^2-x-6} - \frac{5}{x^2-8x+15} \quad \text{LCD: } (x-3)(x+2)(x-5)$$

$$\frac{3x}{(x-3)(x+2)} - \frac{5}{(x-5)(x-3)}$$

$$\frac{(x-5)}{(x-5)} \cdot \frac{3x}{(x-3)(x+2)} + \frac{-5}{(x-5)(x-3)} \cdot \frac{(x+2)}{(x+2)}$$

$$\frac{3x^2-15x}{(x-3)(x+2)(x-5)} + \frac{-5x-10}{(x-3)(x+2)(x-5)}$$

$$\boxed{\frac{3x^2-20x-10}{(x-3)(x+2)(x-5)}}$$

Solve each rational "work" problem.

23) Suppose one painter can paint an entire house in twelve hours and a second painter takes 8 hours to complete the same house. How long would it take the two painters if they worked together?

First: 1 house in 12 hrs

$$\frac{1}{12} \text{ house/hr}$$

Second: 1 house in 8 hrs

$$\frac{1}{8} \text{ house/hr}$$

$$\frac{1}{12} + \frac{1}{8} = \frac{1}{x} \quad \text{LCD: } 24x$$

$$2x + 3x = 24$$

$$5x = 24$$

$$\boxed{x = 4.8 \text{ hours}}$$

24) A kayaker spends an afternoon paddling on a river. She travels 3 miles upstream and 3 miles downstream in a total of 4 hours. In still water, the kayaker can travel an average speed of 2 miles/hour. Based on this information, what is the average speed of the river's current?

upstream: 2 - current

$$\frac{3}{2-c}$$

downstream: 2 + current

$$\frac{3}{2+c}$$

$$\frac{3}{2-c} + \frac{3}{2+c} = 4 \quad \text{LCD: } (2-c)(2+c) = 4-c^2$$

$$3(2+c) + 3(2-c) = 4(4-c^2)$$

$$6+3c+6-3c = 16-4c^2$$

$$-4 = -4c^2$$

$$1 = c^2$$

$$\text{current} = 1 \text{ mile/hr}$$