REVIEW Solving Polynomials Name: Answer Key

1. a. Write an expression that represents the area of the top face of a cylinder when the height is $x + 2$ and the volume of the cylinder is $x^3 - x^2 - 6x$. $A = \frac{V}{h}$

$$\begin{array}{r|cccc} -2 & 1 & -1 & -6 & 0 \\ & \downarrow & -2 & 6 & 0 \\ \hline & 1 & -3 & 0 & 0 \end{array}$$

$A = x^2 - 3x$

b. Evaluate the ~~volume~~ area for $A(6)$ and $A(10.2)$

$A(6) = (6)^2 - 3(6)$

$A(6) = 18$

$A(10.2) = (10.2)^2 - 3(10.2)$

$A(10.2) = 73.44$

2. Determine whether the given binomial is a factor of the polynomial $P(x)$. Also, explain the meaning of the remainder.

a. $(x + 2)$; $(x^2 + 5x + 6)$

$$\begin{array}{r|ccc} -2 & 1 & 5 & 6 \\ & & -2 & -6 \\ \hline & 1 & 3 & 0 \end{array}$$

$x+2$ is a factor because the remainder equals zero.

b. ~~(x+2)~~ $(2x-3)$; $(3x^4 - 6x^3 - 30)$ $(2x-3) \rightarrow (x - \frac{3}{2})$

$$\begin{array}{r|ccccc} \frac{3}{2} & 3 & -6 & 0 & 0 & -30 \\ & \downarrow & 4.5 & -2.25 & -3.375 & -5.0625 \\ \hline & 3 & -1.5 & -2.25 & -3.375 & -35.0625 \end{array}$$

$(2x-3)$ is not a factor because the remainder is not zero

A remainder of zero means that an expression evenly divides into a polynomial.

3. Write the factored form of the simplest function with zeros $2i$, $-2i$ *and* -6. What is the degree of the polynomial?

$(x-2i)(x+2i)(x+6)$ Degree: 3

4. Write the factored form of the simplest function with zeros $-3i$, $6i$, and 5. What is the degree of the polynomial?

$(x+3i)(x-3i)(x-6i)(x+6i)(x-5)$ Degree: 5

Imaginary roots will always have its opposite sign pair

5. Solve the following polynomials by finding the linear or quadratic factors and the zeros. Use factoring methods, synthetic division, quadratic formula and graphing as needed.

a. $2x^3 + x^2 + 8x + 4 = 0$

$x^2(2x+1) + 4(2x+1)$

$(x^2+4)(2x+1) = 0$

$x^2+4=0 \quad 2x+1=0$

$x^2 = -4 \quad 2x = -1$

$x = \pm 2i \quad x = -1/2$

b. $64x^3 - 8 = 0$

$8(8x^3 - 1) = 0$ ← diff. of cubes

$8(2x-1)(4x^2+2x+1) = 0$

$4x^2+2x+1=0 \qquad 2x-1=0$

$\frac{-2 \pm \sqrt{2^2-4(4)(1)}}{2(4)} \qquad 2x = 1$

$x = 1/2$

$\frac{-2 \pm \sqrt{-12}}{8}$

$\frac{-2 \pm 3.464i}{8} \rightarrow$

$x = -.25 + .433i$

$x = -.25 - .433i$

c. $11x^3 - 7x - 4 = 0$

Real root: $x = 1 \rightarrow (x-1)$

1	11	0	-7	-4
	↓	11	11	4
	11	11	4	0

$11x^2 + 11x + 4 = 0$

$\frac{-11 \pm \sqrt{11^2 - 4(11)(4)}}{2(11)}$

$\frac{-11 \pm \sqrt{-55}}{22}$

$\frac{-11 \pm 7.416i}{22} \rightarrow$

$x = 1$

$x = -.5 + .337i$

$x = -.5 - .337i$

d. $x^4 + x^3 + 7x^2 + 9x = 18$

$x^4 + x^3 + 7x^2 + 9x - 18 = 0$

Real roots: $x = -2$ $(x+2)$ and $x = 1$ $(x-1)$

-2	1	1	7	9	-18
	↓	-2	2	-18	18
	1	-1	9	-9	0

$x^3 - x^2 + 9x - 9$

1	1	-1	9	-9
	↓	1	0	9
	1	0	9	0

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

$x = -2 \quad x = 1$

$x = 3i \quad x = -3i$