Solving 3x3 Systems

Solve the system.

$2x - 4y + z = 10$

$x + 2y - z = 1$

$-x - 3y + 2z = 0$

Step 1: Select an equation and solve it for a variable. The 1st or 2nd would be good choices because one of their variables has a coefficient of +1. Let's solve the second equation for x.

$x + 2y - z = 1$

$x = -2y + z + 1$

Step 2: Substitute this equation into the other two equations. Simplify.

$2x – 4y + z = 10$

$2(-2y + Z + 1) – 4y + z = 10$

$-4y + 2z + 2 – 4y + z = 10$

$-8y + 3z + 2 = 10$

$-8y + 3z = 8$

$-x – 3y + 2z = 0$

$-(-2y + Z + 1) – 3y + 2z = 0$

$2y – z – 1 – 3y + 2z = 0$

$-y + z – 1 = 0$

$-y + z = 1$

Now we have a system of two equations with two variables.

$-8y + 3z = 8$

$-y + z = 1$

Step 3: Solve one of the equations for a variable. Choose the 2nd and solve for z.

$-y + z = 1$

$z = y + 1$

Step 4: Substitute this equation into the other. Solve for the variable.

$-8y + 3z = 8$

$-8y + 3(y + 1) = 8$

$-8y + 3y + 3 = 8$

$-5y + 3 = 8$

$-5y = 5$

$y = -1$

Step 5: Substitute the value of y into the equation for z.

$z = y + 1$

$z = (-1) + 1$

$z = 0$

Step 6: Substitute the values of y and z into the equation for x.

$x = -2y + z + 1$

$x = -2(-1) + 0 + 1$

$x = 2 + 0 + 1$

$x = 3$

The solution to the system is:

$x = 3, \qquad y = -1, \qquad z = 0$

As an order triple: $(3, -1, 0)$

Solve the system.

$3x - y + z = -1$

$2x + 3y + z = 4$

$5x + 4y + 2z = 5$

Step 1: Select an equation and solve it for a variable. Let's solve the first equation for z.

$3x - y + z = -1$

$z = -3x + y - 1$

Step 2: Substitute this equation into the other two equations. Simplify.

$2x + 3y + z = 4$

$2x + 3y + (-3x + y - 1) = 4$

$2x + 3y - 3x + y - 1 = 4$

$-x + 4y - 1 = 4$

$-x + 4y = 5$

$5x + 4y + 2z = 5$

$5x + 4y + 2(-3x + y - 1) = 5$

$5x + 4y - 6x + 2y - 2 = 5$

$-x + 6y - 2 = 5$

$-x + 6y = 7$

Now we have a system of two equations with two variables.

$-x + 4y = 5$

$-x + 6y = 7$

Step 3: Solve one of the equations for a variable. Choose the 1st and solve for x.

$-x + 4y = 5$

$-x = -4y + 5$

$x = 4y - 5$

Step 4: Substitute this equation into the other. Solve for the variable.

$-x + 6y = 7$

$-(4y - 5) + 6y = 7$

$-4y + 5 + 6y = 7$

$2y + 5 = 7$

$2y = 2$

$y = 1$

Step 5: Substitute the value of y into the equation for x.

$x = 4y – 5$

$x = 4(1) – 5$

$x = 4 – 5$

$x = -1$

Step 6: Substitute the values of x and y into the equation for z.

$z = -3x + y – 1$

$z = -3(-1) + (1) – 1$

$z = 3 + 1 – 1$

$z = 3$

The solution to the system is:

$x = -1, \qquad y = 1, \qquad z = 3$

As an order triple: $(-1, 1, 3)$

Solve the system.

$3x - y + 4z = -10$

$-x + y + 2z = 6$

$2x - y + z = -8$

Step 1: Select an equation and solve it for a variable. Any equation would work because they all have a variable with a coefficient of 1. Let's choose the 2^{nd} and solve for y.

$-x + y + 2z = 6$

$y = x - 2z + 6$

Step 2: Substitute this equation into the other two equations. Simplify.

$3x – y + 4z = -10$

$3x – (x – 2z + 6) + 4z = -10$

$3x – x + 2z – 6 + 4z = -10$

$2x + 6z = -4$

$2x – y + z = -8$

$2x – (x – 2z + 6) + z = -8$

$2x – x + 2z – 6 + z = -8$

$x + 3z = -2$

Now we have a system of two equations with two variables.

$2x + 6z = -4$

$x + 3z = -2$

Step 3: Solve one of the equations for a variable. Choose the 2^{nd} and solve for x.

$x + 3z = -2$

$x = -3z - 2$

Step 4: Substitute this equation into the other. Solve for the variable.

$2x + 6z = -4$

$2(-3z - 2) + 6z = -4$

$-6z - 4 + 6z = -4$

$-4 = -4$

This equation is always true. This means that the value of z does not impact the solution. It could be literally any and every number. Therefore, there are infinitely many solutions.