Solving Absolute Value Equations

When solving absolute value functions, you must consider two cases:

The absolute value is positive and the absolute value is negative.

This is because $|x| = 5$ when $x = 5$ and $x = -5$.

Positive case:

Drop abs. value bars

Negative case:

Drop abs. value bars,

Negate opp. side of =

Solve: $|x + 3| = 7$

Case 1:

$x + 3 = 7$

$x = 4$ Subtract 3

Case 2:

$x + 3 = -(7)$

$x + 3 = -7$

$x = -10$ Subtract 3

<u>Substitute to check your answers</u>

$|4 + 3| = 7$

$|7| = 7$

$7 = 7$

$|-10 + 3| = 7$

$|-7| = 7$

$7 = 7$

Solve: $|2x - 10| = -3$

Case 1:

$2x - 10 = -3$

$2x = 7$ Add 10

$x = 3.5$ Divide by 2

Case 2:

$2x - 10 = -(-3)$

$2x - 10 = 3$

$2x = 13$ Add 10

$x = 6.5$ Divide by 2

Substitute to check your answers

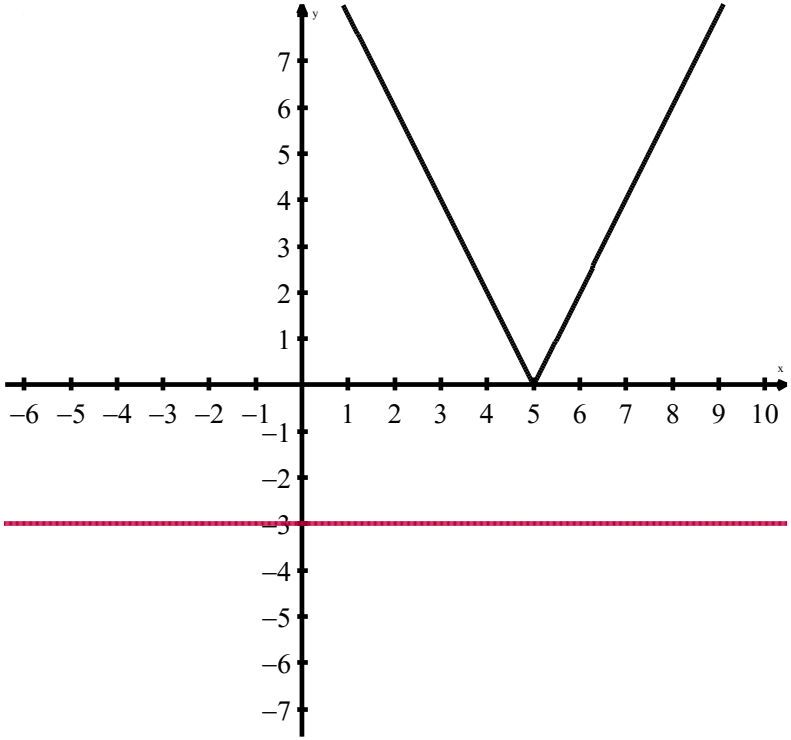
$$|2(3.5) - 10| = -3$$
$$|-3| = -3$$
$$3 \neq -3$$

$$|2(6.5) - 10| = -3$$
$$|3| = -3$$
$$3 \neq -3$$

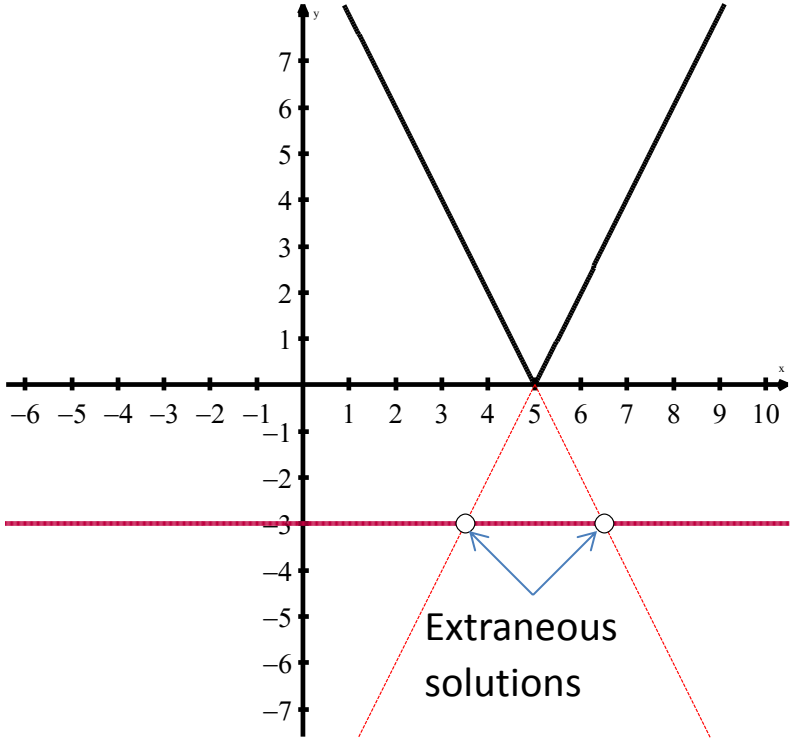
Both answers are extraneous.
There is no solution.

Extraneous solution: A solution you get when solving that does not work when substituted back in. Extraneous solutions are not solutions to the equation.

As you can see, the abs. value graph does not intersect the line y = -3. Hence, no solution.



You find the extraneous solutions by extending the sides of the graph



Solve: $2|x + 5| - 1 = 11$

$2|x + 5| = 12$

$|x + 5| = 6$

Isolate the absolute value before doing the two cases.

Case 1:

$x + 5 = 6$

$x = 1$ Subtract 5

$2|1 + 5| - 1 = 11$

$2|6| - 1 = 11$

$11 = 11$

$x = 1$ is a solution

Case 2:

$x + 5 = -6$

$x = -11$ Subtract 5

$2|-11 + 5| - 1 = 11$

$2|-6| - 1 = 11$

$11 = 11$

$x = -11$ is a solution

Solve: $\frac{1}{2}|4x + 2| = 8x + 9$

$|4x + 2| = 2(8x + 9)$

$|4x + 2| = 16x + 18$

Isolate the absolute value before doing the two cases.

Case 1:

$4x + 2 = 16x + 18$

$-16 = 12x$ Group like terms

$x = -1.\overline{3}$ Divide

$\frac{1}{2}|4(-1.\overline{3}) + 2| = 8(-1.\overline{3}) + 9$

$\frac{1}{2}|-3.\overline{3}| = -10.\overline{6} + 9$

$1.\overline{6} = -1.\overline{6}$

$x = -1.\overline{3}$ is extraneous.

Case 2:

$4x + 2 = -(16x + 18)$ Negate the right side.

$4x + 2 = -16x - 18$

$20x = -20$ Group like terms

$x = -1$ Divide

$\frac{1}{2}|4(-1) + 2| = 8(-1) + 9$

$\frac{1}{2}|-2| = -8 + 9$

$1 = 1$

$x = 1$ is the solution

Solve: $3|x - 5| = 3x + 6$

$|x - 5| = x + 2$

Isolate the absolute value before doing the two cases.

Case 1:

$x - 5 = x + 2$

$-5 = 2$ The x's cancel

DNE -5 does not equal 2, parallel

Case 2:

$x - 5 = -(x + 2)$

$x - 5 = -x - 2$ Negate the right side

$2x = 3$ Group like terms

$x = 1.5$ Divide

$3|1.5 - 5| = 3(1.5) + 6$

$3|-3.5| = 4.5 + 6$

$10.5 = 10.5$

$x = 1.5$ is the solution.