Solving Polynomials by Factoring

$$4x^5 + 4x^4 - 24x^3 = 0$$

$4x^3(x^2 + x - 6) = 0$ *Factor out the GCF, $4x^4$.*

$4x^3(x + 3)(x - 2) = 0$ *Factor the quadratic.*

$4x^3 = 0$ or $(x + 3) = 0$ or $(x - 2) = 0$ *Set each factor equal to 0.*

$x = 0, x = -3, x = 2$ *Solve for x.*

x = 0 is a triple root (multiplicity of 3);
–3 and 2 are both single roots.

Solve the polynomial equation by factoring.

$2x^3 - 20x^2 = -50x$

$2x^3 - 20x^2 + 50x = 0$ *Write in standard form.*

$2x(x^2 - 10x + 25) = 0$ *Factor out the GCF, $2x^4$.*

$2x(x - 5)(x - 5) = 0$ *Factor the quadratic.*

$2x = 0$ or $(x - 5)^2 = 0$ *Set each factor equal to 0.*

$x = 0, x = 5$ *Solve for x.*

x = 5 is a double root and x = 0 is a single root.

Solve the polynomial equation by factoring.

$$x^3 - 2x^2 - 25x = -50$$

$x^3 - 2x^2 - 25x + 50 = 0$ *Set the equation equal to 0.*

$x^2(x - 2) - 25(x - 2) = 0$ *Factor by grouping.*

$(x^2 - 25)(x - 2) = 0$

$(x + 5)(x - 5)(x - 2) = 0$ *Factor by diff of squares.*

$x + 5 = 0$, $x - 5 = 0$, or $x - 2 = 0$

The roots are –5, 5, and 2.

Solve the polynomial equation by factoring.

$x^3 + 3x^2 - 7x = 0$

$x(x^2 + 3x - 7) = 0$

Factor the GCF; what is inside the parenthesis cannot factor. Use quadratic formula.

$1x^2 + 3x - 7 = 0$

Set the parenthesis = 0.

$b^2 - 4ac$

Evaluate the discriminant

$(3)^2 - 4(1)(-7)$

Discriminant = 37

$$x = \frac{-3 \pm \sqrt{37}}{2}$$

Sub into the quadratic formula.

$$x = \frac{-3 + \sqrt{37}}{2} \qquad x = \frac{-3 - \sqrt{37}}{2}$$

Write as two answers: one plus, one minus. Simplify.

$$x \approx 1.541 \qquad x \approx -4.541$$

The zeros are x = -4.541, x = 1.541, and x = 0 (← from the GCF).

Write a polynomial that has a zero at 4 with double multiplicity, a zero at -1, and a zero at zero.

Mult. of 2	Mult. of 1	Mult. of 1
$(x - 4)^2$	$(x + 1)$	(x)
Zero at 4	Zero at -1	Zero at 0

$P(x) = (x - 4)^2(x + 1)(x)$ Multiply the factors

This polynomial has a degree of 4

Add the exponents: 2 + 1 + 1 = 4.

Write a polynomial that has a zero at 5 multiplicity of 2, a zero at 3, a zero at -3 with multiplicity 3, and a zero at zero.

Mult. of 2	Mult. of 1	Mult. of 3	Mult. of 1
$(x - 5)^2$	$(x - 3)$	$(x + 3)^3$	(x)
Zero at 5	Zero at 3	Zero at -3	Zero at 0

$P(x) = (x - 5)^2(x - 3)(x + 3)^3(x)$ Multiply the factors

This polynomial has a degree of 7

Add the exponents: 2 + 1 + 3 + 1 = 7.