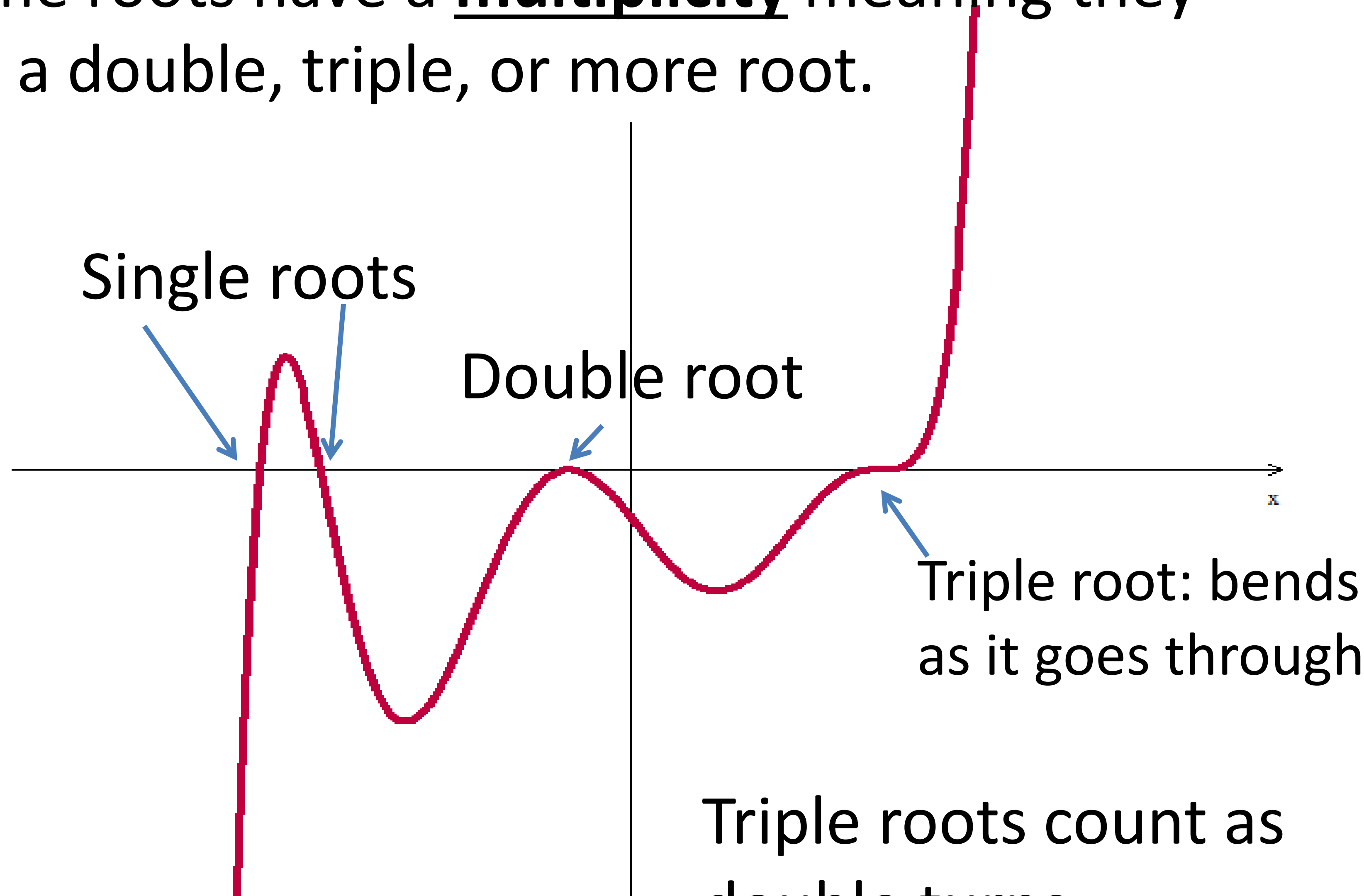
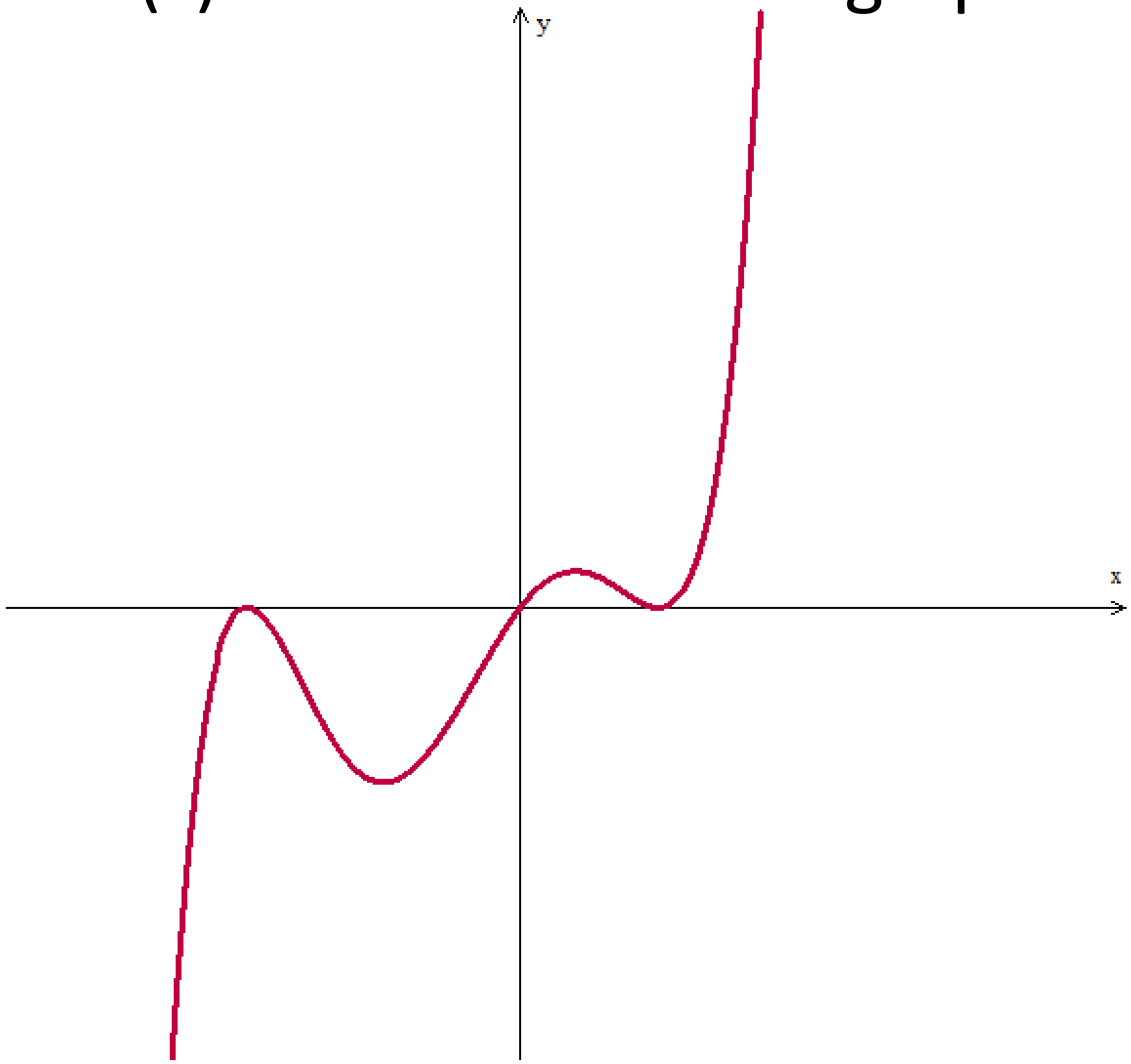
Solving Polynomials with Complex Roots

Some roots have a **multiplicity** meaning they are a double, triple, or more root.



Triple roots count as double turns.

What kind(s) of roots does this graph have?



This graph has 2 double roots and 1 single root, 4 turns. It is degree 5.

Determine the degree of the polynomial from its graph.

This graph has 1 double root and 2 single roots, it has 5 turns. So it must be at least degree 6 based on the turns.

If the polynomial is degree 6 but only has 4 roots accounted for (1 double, 2 single), then where are the other two roots? They're imaginary!

The degree of a polynomial tells you how many zeroes it will have. A 4^{th} degree will always have 4 roots.

Some of those 4 zeroes may be imaginary.

Imaginary solutions always come in pairs.
(i.e. +4i and -4i or 3+2i and 3–2i)

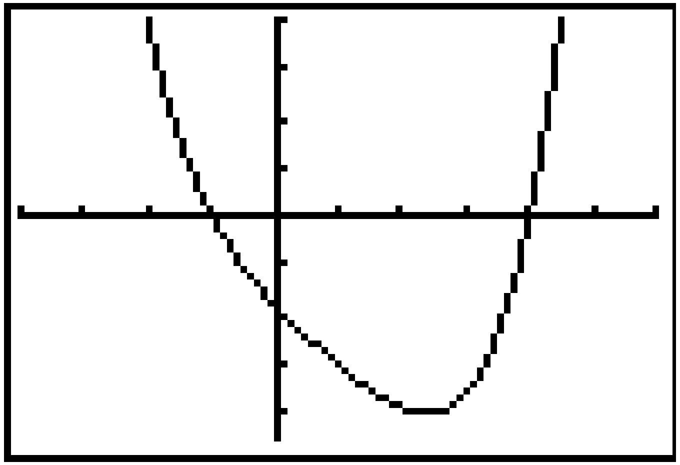
A 4^{th} degree polynomial could have: 4 real roots, 2 real and 2 imaginary, or 4 imaginary.

Use the graphing, synthetic division, and (possibly) the quadratic formula to solve.

Solve $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$ by finding all roots.

The polynomial is of degree 4, so there are exactly four roots for the equation.

Step 1 Graph the function to find all real roots



Find the real roots at −1 and 4.

Step 2 Use synthetic division to simplify the polynomial.

−1	1	−3	5	−27	−36
		−1	4	−9	36
	1	−4	9	−36	0

Use x = -1, the factor (x + 1), for synthetic division.

The simplified polynomial is now: $(x + 1)(x^3 - 4x^2 + 9x - 36)$

Step 3 Use synthetic division to simplify the polynomial with the other root.

4	1	−4	9	−36
		4	0	36
	1	0	9	0

Use x = 4, the factor (x - 4), for synthetic division.

The simplified polynomial is now: $(x + 1)(x - 4)(x^2 + 9)$

Step 4 Solve $x^2 + 9 = 0$ to find the remaining roots.

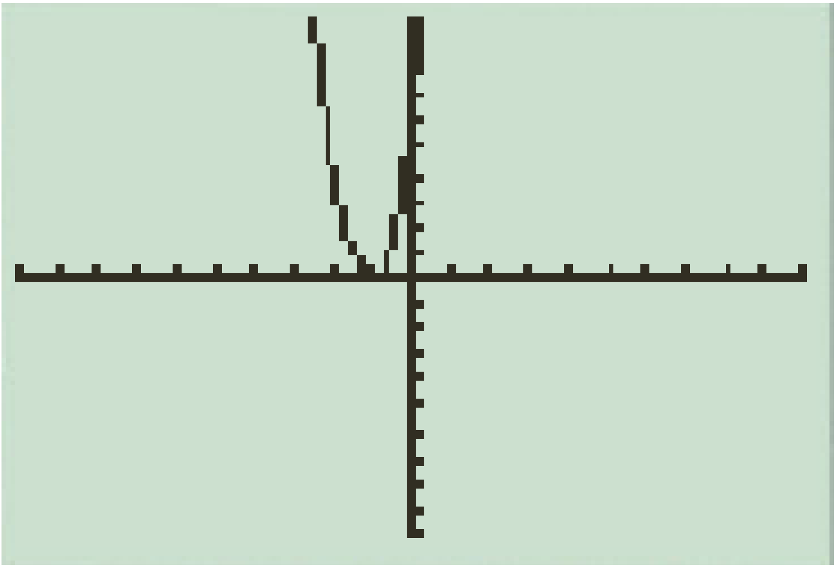
$$x^2 + 9 = 0$$
$$x^2 = -9$$
$$x = \pm 3i$$

The fully factored form of the equation is $(x + 1)(x - 4)(x + 3i)(x - 3i) = 0$. The solutions are $4, -1, 3i, -3i$.

Solve $x^4 + 5x^3 + 13x^2 + 15x + 6 = 0$ by finding all roots.

The polynomial is of degree 4, so there are exactly four roots for the equation.

Step 1 Graph the function to find all real roots



There is a double root at x = -1

Step 2 Use synthetic division to simplify the polynomial.

-1	1	5	13	15	6
		-1	-4	-9	-6
	1	4	9	6	0

Use x = -1 , the factor (x + 1), for synthetic division.

The simplified polynomial is now: $(x + 1)(x^3 + 4x^2 + 9x + 6)$

Step 3 Use synthetic division to simplify the polynomial with the other root.

-1	1	4	9	6
		-1	-3	-6
	1	3	6	0

Use x = -1 , the factor (x + 1), for synthetic division (again)

The simplified polynomial is now: $(x + 1)(x + 1)(x^2 + 3x + 6)$

Step 4 Solve $x^2 + 3x + 6 = 0$ to find the remaining roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Write the Quadratic Formula.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(6)}}{2(1)}$$

Substitute and simplify.

$$x = \frac{-3 \pm \sqrt{9 - 24}}{2}$$

Write as 2 solutions.

$$x = \frac{-3 + \sqrt{-15}}{2} \qquad x = \frac{-3 - \sqrt{-15}}{2}$$

Simplify.

$$x = -1.5 + 1.936i \qquad x = -1.5 - 1.936i$$

The fully factored form of the equation is

$(x + 1)(x + 1)(x + (-1.5 + 1.936i))(x - (-1.5 - 1.936i)) = 0.$

The solutions are -1 mult. of 2, $-1.5 + 1.936i$, $-1.5 - 1.936i$