

Solving Rational Equations

A rational function is **undefined** at holes and vertical asymptotes.

If you find a solution where the function is undefined, then that solution is **extraneous**.

Steps:

- 1) Factor and identify the LCD.
- 2) Multiply each term by the LCD and simplify.
- 3) Solve the remaining equation.

Solve: $\frac{2x - 5}{x - 8} + \frac{x}{2} = \frac{11}{x - 8}$

LCD: $2(x - 8)$ Identify the LCD

$2(x - 8) \frac{2x - 5}{(x - 8)} + 2(x - 8) \frac{x}{2} = \frac{11}{(x - 8)} 2(x - 8)$ Multiply each term by the LCD.

~~$2(x - 8) \frac{2x - 5}{(x - 8)} + 2(x - 8) \frac{x}{2} = \frac{11}{(x - 8)} 2(x - 8)$~~ Cancel common factors before simplifying.

$2(2x - 5) + x(x - 8) = 11(2)$

Multiply the remaining terms.

$4x - 10 + x^2 - 8x = 22$

Solve the resulting equation.

$x^2 - 4x - 32 = 0$

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$$(x - 8)(x + 4) = 0$$

$$x - 8 = 0 \text{ and } x + 4 = 0$$

$$x = 8 \text{ and } x = -4$$

$x = 4$ is the solution.

$x = 8$ is extraneous because the function is undefined at $x = 8$ (vertical asymptote)

Solve: $\frac{16}{x^2 - 16} = \frac{3}{x - 4}$

$$\frac{16}{(x + 4)(x - 4)} = \frac{3}{(x - 4)}$$

Factor

LCD: $(x + 4)(x - 4)$ Identify the LCD

$$(x + 4)(x - 4) \frac{16}{(x + 4)(x - 4)} = \frac{3}{(x - 4)} (x + 4)(x - 4)$$

Multiply each term by the LCD.

$$\cancel{(x + 4)} \cancel{(x - 4)} \frac{16}{\cancel{(x + 4)} \cancel{(x - 4)}} = \frac{3}{\cancel{(x - 4)}} \cancel{(x + 4)} \cancel{(x - 4)}$$

Cancel common factors before simplifying.

$$16 = 3(x + 4)$$

Multiply the remaining terms.

$$16 = 3x + 12$$

Solve the resulting equation.

$$16 = 3x + 12$$

$$4 = 3x$$

$$x = \frac{3}{4}$$

$x = \frac{3}{4}$ is the solution. There are no extraneous solutions because the function is only undefined at $x = 4$ and $x = -4$

Solve: $\frac{4}{x+2} + \frac{5}{x-2} = \frac{29}{x^2-4}$

$$\frac{4}{x+2} + \frac{5}{x-2} = \frac{29}{(x+2)(x-2)} \quad \text{Factor}$$

LCD: $(x+2)(x-2)$ Identify the LCD

Multiply each term by the LCD.

$$(x+2)(x-2) \frac{4}{(x+2)} + (x+2)(x-2) \frac{5}{(x-2)} = \frac{29}{(x+2)(x-2)} (x+2)(x-2)$$

Cancel common factors before simplifying.

$$\cancel{(x+2)}\cancel{(x-2)} \frac{4}{\cancel{(x+2)}} + \cancel{(x+2)}\cancel{(x-2)} \frac{5}{\cancel{(x-2)}} = \frac{29}{\cancel{(x+2)}\cancel{(x-2)}} \cancel{(x+2)}\cancel{(x-2)}$$

$$4(x-2) + 5(x+2) = 29$$

Multiply the remaining terms.

$$4x - 8 + 5x + 10 = 29$$

Solve the resulting equation.

$$9x + 2 = 29$$

$$9x = 27$$

$$x = 3$$

The solution is $x = 3$. There are no extraneous solutions.