

Solving Rational Inequalities

Steps:

- 1) Solve the inequality like an equation to find the zeros.
- 2) Find where the rational function is undefined (holes, VA)
- 3) Set up a chart with the zeros/undefined values and fill it in.
- 4) Write your answer using the chart and inequality.

Solve $\frac{6}{x-8} \leq 3$

Step 1: Solve as an equation.

$$(x - 8) \frac{6}{x - 8} = 3(x - 8)$$

$$6 = 3x - 24$$

$$30 = 3x$$

$$x = 10$$

Step 2: Find where the function is undefined.

Vertical Asymptote

$$x - 8 = 0$$

$$x = 8$$

Step 3: Set up and fill in a chart.

x		8		10	
f(x)		DNE		0	

Pick numbers to fill-in the spaces.

$$\frac{6}{x-8} \leq 3$$

x	6	8	9	10	11
f(x)	True	DNE	False	0	True

$$f(6) = \frac{6}{(6)-8} \leq 3 \rightarrow -3 \leq 3 \text{ True}$$

$$f(9) = \frac{6}{(9)-8} \leq 3 \rightarrow 6 \leq 3 \text{ False}$$

$$f(11) = \frac{6}{(11)-8} \leq 3 \rightarrow 2 \leq 3 \text{ True}$$

Step 4: Write your answer using the chart.

x	6	8	9	10	11
f(x)	True	DNE	False	0	True

$$\frac{6}{x-8} \leq 3$$

Solution: $(-\infty, 8) \cup [10, \infty)$

Do not include 8 because $x = 8$ is undefined.

Include 10 because the inequality is less than or equal to.

Solve $\frac{10}{x-5} \geq \frac{11}{x-6}$

Step 1: Solve as an equation.

$$(x-6)(x-5) \frac{10}{x-5} = \frac{11}{x-6} (x-6)(x-5)$$

$$10(x-6) = 11(x-5)$$

$$10x - 60 = 11x - 55$$

$$-5 = x$$

Step 2: Find where the function is undefined.

Vertical Asymptotes

$$x - 5 = 0 \text{ and } x - 6 = 0$$

$$x = 5 \text{ and } x = 6$$

Step 3: Set up and fill in a chart.

x		-5		5		6	
f(x)		0		DNE		DNE	

Pick numbers to fill-in the spaces.

x	-6	-5	0	5	5.5	6	7
f(x)	True	0	False	DNE	True	DNE	False

$$\frac{10}{x-5} \geq \frac{11}{x-6}$$

$$f(-6) = \frac{10}{(-6-5)} \geq \frac{11}{(-6-6)} \rightarrow \frac{10}{-11} \geq \frac{11}{-12} \quad \text{True}$$

$$f(0) = \frac{10}{(0-5)} \geq \frac{11}{(0-6)} \rightarrow \frac{10}{-5} \geq \frac{11}{-6} \quad \text{False}$$

$$f(5.5) = \frac{10}{(5.5-5)} \geq \frac{11}{(5.5-6)} \rightarrow \frac{10}{.5} \geq \frac{11}{-.5} \quad \text{True}$$

$$f(7) = \frac{10}{(7-5)} \geq \frac{11}{(7-6)} \rightarrow \frac{10}{2} \geq \frac{11}{1} \quad \text{False}$$

Step 5: Write your answer using the sign chart and inequality.

x	-6	-5	0	5	5.5	6	7
f(x)	True	0	False	DNE	True	DNE	False

$$\frac{10}{x-5} \geq \frac{11}{x-6}$$

Solution: $(-\infty, -6] \cup (5, 6)$

Do not include 5 and 6 because they are undefined.

Include -6 because the inequality is greater than or equal to.