

Solving Square Root Equations

Solve the equation $5 + \sqrt{x + 1} = 16$

$$5 + \sqrt{x + 1} = 16$$

$$\sqrt{x + 1} = 16 - 5$$

Subtract 5.

$$\sqrt{x + 1} = 11$$

Simplify.

$$(\sqrt{x + 1})^2 = (11)^2$$

Square both sides.

$$x + 1 = 121$$

Simplify.

$$x = 120$$

Solve for x.

Check

$$5 + \sqrt{x + 1} = 16$$

$$5 + \sqrt{120 + 1} = 16$$

$$5 + \sqrt{121} = 16$$

$$16 = 16 \quad \checkmark$$

Solve the equation $\sqrt{2x - 3} + 12 = 18$

$$\sqrt{2x - 3} + 12 = 18$$

$$\sqrt{2x - 3} = 18 - 12 \quad \text{Subtract 12.}$$

$$\sqrt{2x - 3} = 6 \quad \text{Simplify.}$$

$$\sqrt{2x - 3}^2 = 6^2 \quad \text{Square both sides.}$$

$$2x - 3 = 36 \quad \text{Simplify.}$$

$$2x = 39 \quad \text{Solve for } x.$$

$$x = 19.5$$

Check

$\sqrt{2x - 3} + 12 = 18$	
$\sqrt{2(19.5) - 3} + 12$	18
$\sqrt{39 - 3} + 12$	18
$\sqrt{36} + 12$	18
$6 + 12$	18
$\checkmark 18$	18

Solve the equation $\sqrt{7x + 2} = 3\sqrt{3x - 2}$

$$\sqrt{7x + 2} = 3\sqrt{3x - 2}$$

$$\left(\sqrt{7x + 2}\right)^2 = \left(3\sqrt{3x - 2}\right)^2$$

Square both sides.

$$7x + 2 = 9(3x - 2)$$

Simplify.

$$7x + 2 = 27x - 18$$

Distribute.

$$20 = 20x$$

Solve for x.

$$1 = x$$

Solve the equation $\sqrt{-3x + 33} = 5 - x$

$$\sqrt{-3x + 33} = 5 - x$$

$$\left(\sqrt{-3x + 33}\right)^2 = (5 - x)^2$$

Square both sides.

$$-3x + 33 = 25 - 10x + x^2$$

Simplify.

$$0 = x^2 - 7x - 8$$

Write in standard form.

$$0 = (x - 8)(x + 1)$$

Factor.

$$x - 8 = 0 \text{ or } x + 1 = 0$$

Solve for x.

$$x = 8 \text{ or } x = -1$$

Check your answers

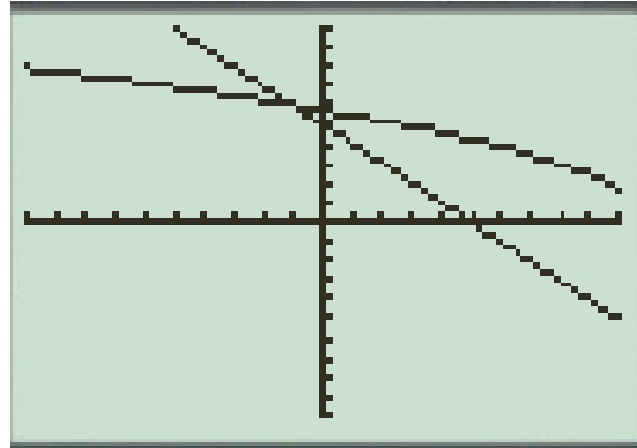
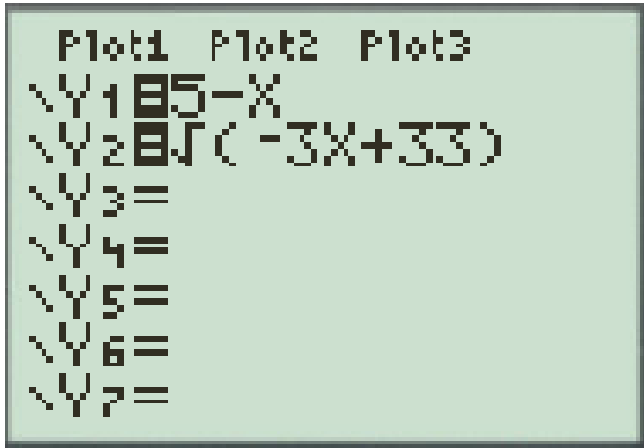
$$\begin{array}{r|l} \sqrt{-3x+33} = 5-x & \\ \hline \sqrt{-3(8)x+33} & 5-8 \\ 3 & -3 \mathbf{x} \end{array}$$

$$\begin{array}{r|l} \sqrt{-3x+33} = 5-x & \\ \hline \sqrt{-3(-1)+33} & 5-(-1) \\ 6 & 6 \checkmark \end{array}$$

The only solution is $x = -1$ because $x = 8$ is extraneous.

Why is $x = 8$ extraneous?

Look at the graph.

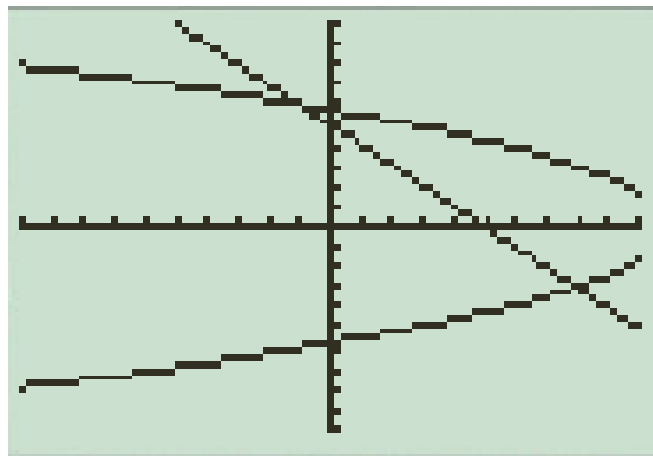


These two graphs intersect when $x = -1$.

What about the $x = 8$?

When we graphed the equations, we only graphed the positive square root. This time include the negative square root.

```
Plot1 Plot2 Plot3
Y1=5-X
Y2=√(-3X+33)
Y3=-√(-3X+33)
Y4=
Y5=
Y6=
Y7=
```



Now we see the extra answer, $x = 8$. It was for the negative square root.

Extraneous solution: A solution that does not solve the original equation.

Solve the equation $\sqrt{7x - 54} - x = -6$

$$\sqrt{7x - 54} = x - 6$$

Isolate the root

$$(\sqrt{7x - 54})^2 = (x - 6)^2$$

Square both sides.

$$7x - 54 = x^2 - 12x + 90$$

Simplify.

$$0 = x^2 - 19x + 90$$

Write in standard form.

$$0 = (x - 10)(x - 9)$$

Factor.

$$x - 10 = 0 \text{ and } x - 9 = 0$$

Solve for x.

$$x = 10 \text{ and } x = 9$$

Check your answers.

$$\sqrt{7x - 54} - x = -6$$

$$\sqrt{7(10) - 54} - 10 \neq -6$$

$$\sqrt{70 - 54} - 10 \neq -6$$

$$-6 \neq -6 \quad \checkmark$$

$$\sqrt{7x - 54} - x = -6$$

$$\sqrt{7(9) - 54} - 9 \neq -6$$

$$\sqrt{63 - 54} - 9 \neq -6$$

$$-6 \neq -6 \quad \checkmark$$

Both $x = 10$ and $x = 9$ are solutions.

Solve the equation $4 + \sqrt{-3m + 10} = m$

$$\sqrt{-3m + 10} = m - 4$$

Isolate the root

$$(\sqrt{-3m + 10})^2 = (m - 4)^2$$

Square both sides.

$$-3m + 10 = m^2 - 8m + 16$$

Simplify.

$$0 = m^2 - 5m + 6$$

Write in standard form.

$$0 = (m - 3)(m - 2)$$

Factor.

$$m - 3 = 0 \text{ and } m - 2 = 0$$

Solve for m.

$$m = 3 \text{ and } m = 2$$

Check your answers.

$$4 + \sqrt{-3m + 10} = m$$

$$4 + \sqrt{-3(3) + 10} = 3$$

$$4 + \sqrt{-9 + 10} = 3$$

$$5 = 3 \quad \mathbf{x}$$

$$4 + \sqrt{-3m + 10} = m$$

$$4 + \sqrt{-3(2) + 10} = 2$$

$$4 + \sqrt{-6 + 10} = 2$$

$$6 = 2 \quad \mathbf{x}$$

Neither $x = 3$ nor $x = 2$ are solutions; they are both extraneous.