Synthetic Division

Synthetic division is a shortcut for polynomial division.

It only works for linear binomials. i.e. $(x + 2)$ $(2x – 3)$

Divide: $(2x^2 + 7x + 9) \div (x + 2)$

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$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ & & & \\ \hline & & & \end{array}$$

Write the coefficients of the dividend (including 0 placeholders if needed)

Use the opposite sign of the number in the divisor.

$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ & & & \\ \hline & 2 & & \end{array}$$

Step 1: Bring down the first coefficient

-2	2	7	9
		-4	
	2		

Step 2: Multiply the divisor with the coefficient. Place it in the next column.
-2 x 2 = -4

-2	2	7	9
		-4	
	2	3	

Step 3: Add 7 + (-4) = 3

-2	2	7	9
		-4	-6
	2	3	

Step 4: Multiply the divisor with the new coefficient. Place it in the next column.

-2 x 3 = -6

-2	2	7	9
		-4	-6
	2	3	3

Step 5: Add 9 + (-6) = 3

This last number is the remainder.

Final answer: put a variable with correct degree on each coefficient.

-2	2	7	9
		-4	-6
	2	3	3

2 ← Degree 1

3 ← Degree 0

3 ← Remainder

Solution: $2x + 3 + \frac{3}{x+2}$

Divide: $(3x^4 - x^3 + 5x - 1) \div (x - 3)$

3	3	-1	0	5	-1

Write the coefficients of the dividend (including 0 placeholders if needed)

Use the opposite sign of the number in the divisor.

3	3	-1	0	5	-1
	3				

Step 1: Bring down the first coefficient

3	3	-1	0	5	-1
		9			
	3				

Step 2: Multiply the divisor with the coefficient. Place it in the next column.
3 x 3 = 9

3	3	-1	0	5	-1
		9			
	3	8			

Step 3: Add -1 + 9 = 8

3	3	-1	0	5	-1
		9	24	72	231
	3	8	24	77	230

Repeat steps 2 and 3 as necessary

3	3	-1	0	5	-1
		9	24	72	231
	3	8	24	77	230

Write final answer with correct degrees.

Degree 3

Degree 2

Degree 1

Degree 0

Remainder

Solution: $3x^3 + 8x^2 + 24x + 77 + \frac{230}{x-3}$

Divide: $(3x^2 + 9x - 2) \div (3x - 1)$

The divisor must have a leading coefficient of 1. Divide it by 3.

Divide: $(3x^2 + 9x - 2) \div (x - 1/3)$

$$
\begin{array}{r|rrr}
1/3 & 3 & 9 & -2 \\
 & & 1 & 3\frac{1}{3} \\
\hline
 & 3 & 10 & 1\frac{1}{3}
\end{array}
$$

Use synthetic division like normal.

Solution: $3x + 10 + \dfrac{1\frac{1}{3}}{x-\frac{1}{3}}$

Note: the remainder is over the new divisor.

Factor Theorem: A binomial is a factor of a polynomial (like (x – 2)(x – 4) are factors of $x^2 - 6x + 8$) if synthetic division has a remainder of zero.

Is (x – 2) a factor of $3x^3 + 2x^2 - 33$?

2	3	2	0	-33
		6	16	32
	3	8	16	-1

(x – 2) is **not** a factor of the polynomial.

Is x = -4 a solution to the polynomial

$P(x) = x^3 + 2x^2 - 3x + 20$?

If x = -4 is a solution, then (x + 4) must be a factor. Use the factor theorem.

-4	1	2	-3	20
		-4	8	-20
	1	-2	5	0

x = -4 is a solution to P(x).

Remainder Theorem: Synthetic division can evaluate a function's value (i.e. f(3)). The answer is the remainder.

Note: you do not use the opposite sign of the #

If $P(x) = 3x^4 - 25x^2 + 4$, find P(-3)

-3	3	0	-25	0	4
		-9	27	-6	18
	3	-9	2	-6	22

P(-3) = 22