Rules for Transformations in Function Notation

A handy chart is provided on the next slide with all of the transformations in function notation.

Rules for Transformations of Functions

If f(x) is the original function, $k > 0$, $h > 0$, $a > 0$, $b > 0$:

Function	**Transformation of the graph of f(x)**
$f(x) + k$	Shift f(x) upward k units
$f(x) - k$	Shift f(x) downward k units
$f(x + h)$	Shift f(x) to the left h units
$f(x - h)$	Shift f(x) to the right h units
$-f(x)$	Reflect f(x) over the x-axis
$f(-x)$	Reflect f(x) over the y-axis
$a \cdot f(x), a > 1$	Stretch f(x) vertically by a factor of a
$a \cdot f(x), 0 < a < 1$	Compress f(x) vertically by a factor of a
$f(bx), b > 1$	Compress f(x) horizontally by a factor of $\frac{1}{b}$
$f(bx), 0 < b < 1$	Stretch f(x) horizontally by a factor of $\frac{1}{b}$

Horizontal shift: $f(x - h)$

Note: Always move the opposite direction of the sign. $f(x + 2)$ makes you think you should move to the right, but you really move left.

Horizontal stretch/compression: $f(bx)$

Note: Always use the reciprocal of the number. For example, $f(2x)$ means $b = \frac{1}{2}$.

Vertical stretches and horizontal compressions have the effect of making the graph narrower.

A vertical stretch pulls the graph away from the x-axis (narrowing).

A vertical compression pushes the graph toward the x-axis (widening).

A horizontal stretch pulls the graph away from the y-axis (widening).

A horizontal compression pushes the graph toward to y-axis (narrowing).

Write each transformation in function notation.

g(x) is shifted up 3 units and vertically compressed by 1/3.

$$\frac{1}{3}g(x) + 3$$

f(x) is shifted right 1 unit and reflected over the x-axis

$$-f(x-1)$$

h(x) is horizontally stretched by 3, shifted to the left 2 units, and shifted down 4.

$$h\left(\frac{1}{3}(x+2)\right) - 4$$

Identify the transformations shown below.

$$-4f(x) + 3$$

Reflect over the x-axis, vertical stretch by 4, shift up 3.

$$g(-5x)$$

Reflect over the y-axis, horizontal compression by $\frac{1}{5}$.

$$h(x + 4) - 2$$

Shift down 2 and shift left 4.