“Work” Problems and Direct/Inverse Variation

Direct variation is a relationship between two variables x and y written in the form: **y = kx**

k is called the **constant of variation**.

If k is positive: as x increases, y increases.

If k is negative: as x increases, y decreases.

(Think in terms of slope)

Given: y varies directly as x, and y = 27 when x = 6. What is x when y = 42?

$y = kx$	*y varies directly as x.*
$27 = k(6)$	*Substitute 27 for y and 6 for x.*
$k = 4.5$	*Solve for the constant of variation k.*
$y = 4.5x$	*Write the variation function by using the value of k.*
$(42) = 4.5x$	*Substitute the 42 for y.*
$x = 9.\overline{3}$	*Simplify.*

Inverse variation is a written: $y = \frac{k}{x}$

If k is positive: as one variable increases, the other decreases.

The time it takes to construct a house varies inversely as the number of workers. 20 workers can build a house in 62.5 hours, how long will it take 15 workers?

$t = \frac{k}{w}$ *t varies directly as w.*

$62.5 = \frac{k}{20}$ *Substitute 62.5 for t and 20 for 2.*

$1250 = k$ *Solve for the constant of variation k.*

$t = \frac{1250}{w}$ *Write the variation function by using the value of k.*

$t = \frac{1250}{15}$ *Substitute the 15 for w.*

$t = 83.\overline{3}$ *Divide.*

"Work" problems come in two different forms:

$$\frac{job}{hours} + \frac{job}{hours} = \frac{total\ job}{total\ hours}$$

or

$$\frac{distance}{rate} + \frac{distance}{rate} = total\ time$$

Kevin can clean a large aquarium tank in about 7 hours. When Kevin and Lara work together, they can clean the tank in 4 hours. Write and solve a rational equation to determine how long, to the nearest tenth of an hour, it would take Lara to clean the tank if she works by herself.

Kevin: 1 tank in 7 hours

1/7 tank in 1 hour

Lara: 1 tank in x hours

1/x tank in 1 hour

Together: 1 tank in 4 hours

1/4 tank in 1 hour

Kevin + Lara = Together

$$\frac{1}{7}+\frac{1}{x}=\frac{1}{4}$$

$$(28x)\left(\frac{1}{7}+\frac{1}{x}=\frac{1}{4}\right)$$

$$4x+28=7x$$

$$28=3x$$

$$x=9.\overline{3}\text{ hours}$$

A riverboat travels at an average of 14 km per hour in still water. The riverboat travels 110 km east up the Ohio River and 110 km down the same river in a total of 17.5 hours. To the nearest tenth of a kilometer per hour, what was the speed of the current of the river?

East: rate = 14 + current

$$time = \frac{110}{14+c}$$

$$Distance = rate \cdot time$$

$$time = \frac{distance}{rate}$$

West: rate = 14 – current

$$time = \frac{110}{14-c}$$

$$time\ going\ East + time\ going\ West = total\ time$$

$$\frac{110}{14+c} + \frac{110}{14-c} = 17.5$$

$$(14+c)(14-c)\left(\frac{110}{14+c} + \frac{110}{14-c} = 17.5\right)$$

$$110(14-c) + 100(14+c) = 17.5(14+c)(14-c)$$

$$1540 - 110c + 1540 + 110c = 17.5(196 - c^2)$$

$$3080 = 3430 - 17.5c^2$$

$$-350 = -17.5c^2$$

$$20 = c^2$$

$$c = 4.472\ {}^{miles}/_{hour}$$