

Writing Rational Functions

The **zeros** (x-intercepts) of a rational function are where the numerator (after simplifying) equals zero.

Find the zeros of: $f(x) = \frac{x^2 - 2x + 1}{x^2 + 8x - 9}$

$$f(x) = \frac{(x-1)(x-1)}{(x+9)(x-1)}$$

Factor the numerator and denominator

$$f(x) = \frac{(x-1)\cancel{(x-1)}}{(x+9)\cancel{(x-1)}}$$

Cancel common factors

$$f(x) = \frac{(x-1)}{(x+9)}$$

Set the numerator equal to 0

$$x - 1 = 0$$

$f(x)$ has a zero at $x = 1$

Find the zeros of: $f(x) = \frac{x^3 + x^2 - 16x - 16}{x^2 - 2x - 3}$

$$f(x) = \frac{(x^2 - 16)(x + 1)}{(x - 3)(x + 1)}$$

Factor the numerator and denominator

$$f(x) = \frac{(x + 4)(x - 4)(x + 1)}{(x - 3)(x + 1)}$$

$$f(x) = \frac{(x + 4)(x - 4)\cancel{(x + 1)}}{(x - 3)\cancel{(x + 1)}}$$

Cancel common factors

$$f(x) = \frac{(x + 4)(x - 4)}{(x - 3)}$$

$x + 4 = 0$ and $x - 4 = 0$ Set the numerator equal to 0

$f(x)$ has zeros at $x = -4$ and $x = 4$

Write the equation of a rational function that:

-has a hole at $x = 4$

-has a vertical asymptote at $x = 2$ and $x = 0$

-has a zero at $x = 3$

-is positive

Hole at $x = 4$

$$\frac{(x-4)}{(x-4)}$$

VA at $x = 2$ and $x = 0$

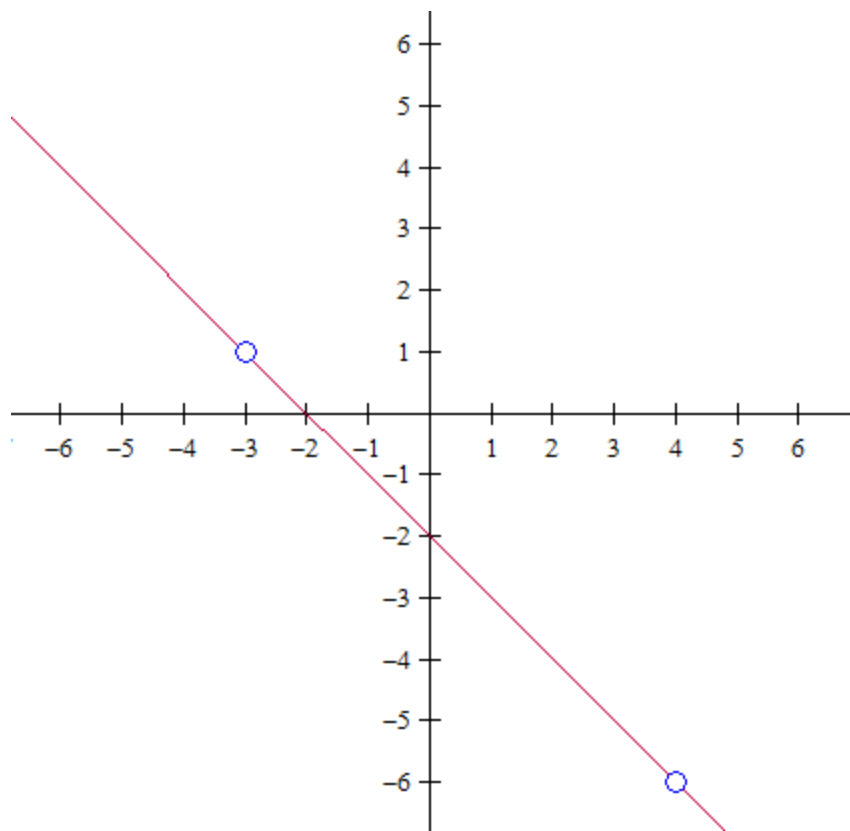
$$\frac{1}{(x)(x-2)}$$

Zero at $x = 3$

$$\frac{(x-3)}{1}$$

$$f(x) = \frac{(x-4)(x-3)}{(x-4)(x)(x-2)}$$

Write an equation for the following graph:



Vertical Asymptote: none

Zero at $x = -2$ $\frac{(x+2)}{1}$

Holes at $x = -3, x = 4$

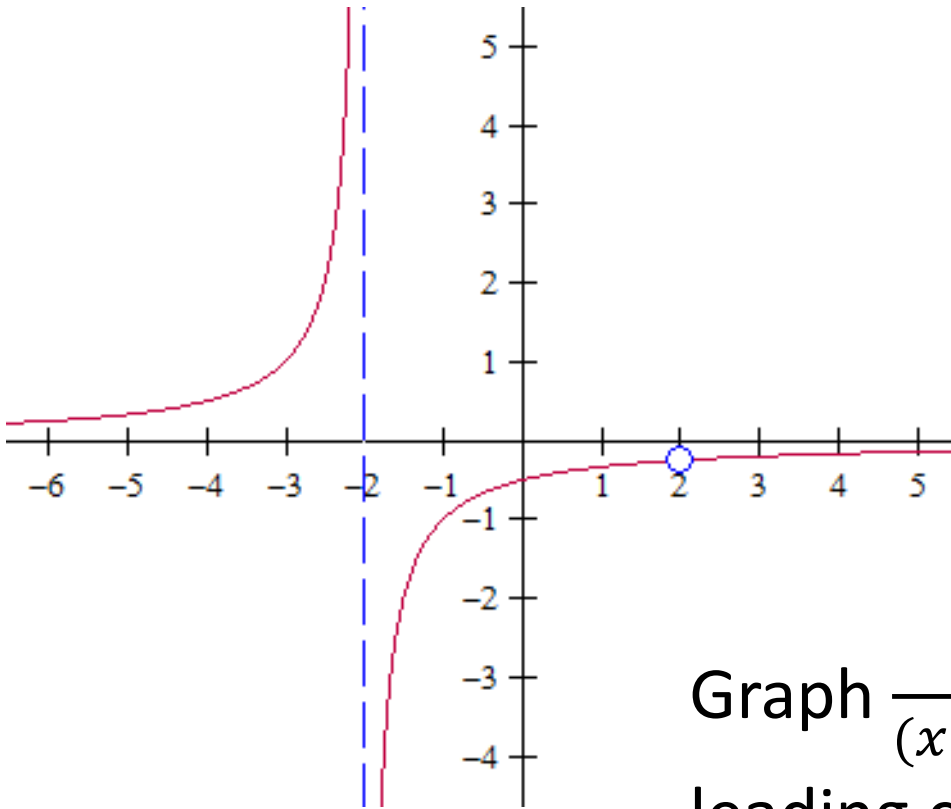
$$\frac{(x+3)(x-4)}{(x+3)(x-4)}$$

Slope of graph: $-1/1$

Equation:

$$f(x) = \frac{-(x+2)(x+3)(x-4)}{(x+3)(x-4)}$$

Write the equation of the following graph:



VA at $x = -2$ $\frac{1}{(x+2)}$

Zeros: none

Hole at $x = 2$ $\frac{(x-2)}{(x-2)}$

Graph $\frac{(x-2)}{(x+1)(x-2)}$ to see that the leading coefficient should be negative.

$$g(x) = \frac{-(x-2)}{(x+2)(x-2)}$$