Calculus Section 1.2 Limits

-Estimate a limit using a numerical or graphical approach.
-Learn different ways that a limit can fail to exist.
-Learn how to write a limit.

Homework: pages 55-58 #'s 2, 5, 15-25, 58, 67-70

The **limit** is the value of f(x) as x approaches a certain number.

A limit is written as: $\lim_{x \to c} f(x) = L$ where:

- lim is the abbreviation of the word limit
- $x \to c$ means that the value of x is approaching a number c
- f(x) is the function
- L is a constant number and the value of the limit

This limit is read: "The limit as x approaches c of f(x) equals L" or "The limit of f(x) as x approaches c of f(x) equals L."

Example: Find the limit of a function numerically.

Find $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

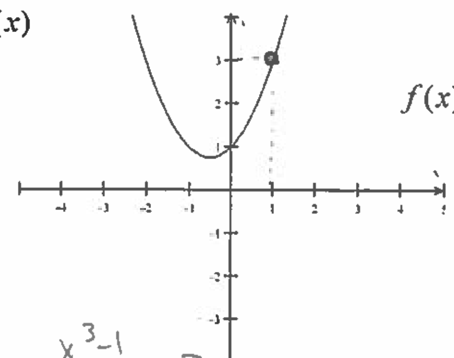
x approaches 1 from the left ⇨ ⇦ x approaches 1 from the right

x	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25
f(x)	2.3125	2.71	2.9701	2.997	DNE	3.003	3.0301	3.31	3.8125

Answer: $\lim_{x \to 1} \frac{x^3-1}{x-1} = 3$

Example: Find the limit of a function graphically.

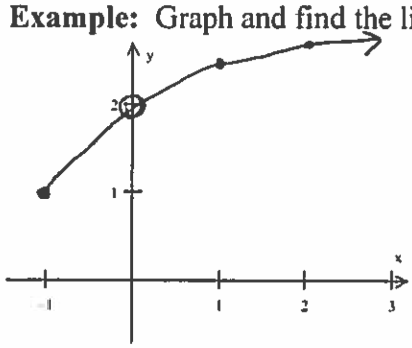
Find $\lim_{x \to 1} f(x)$



$f(x) = \frac{x^3 - 1}{x - 1}$

Answer: $\lim_{x \to 1} \frac{x^3-1}{x-1} = 3$

Example: Graph and find the limit as x approaches zero of the function: $f(x) = \frac{x}{\sqrt{x+1} - 1}$

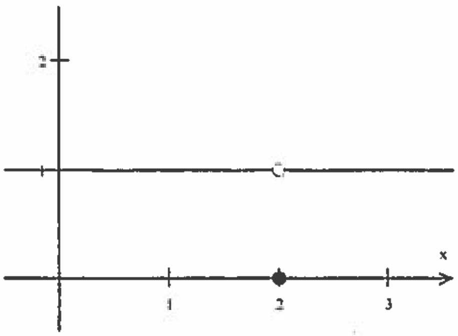


x	-.1	-.01	-.001	0	.001	.01	.1
f(x)	1.949	1.995	1.999	DNE	2.0004	2.005	2.049

Answer: $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1} = 2$

Observation about limit: Eventhough f(0) is undefined, the limit = 2 b/c both sides approach 2

In general, the **limit exists** for a function if the left and the right limits approach the same number. <u>This is regardless of whether the value of f(x) at the limit is equivalent to the limit or exists at all at that point.</u>

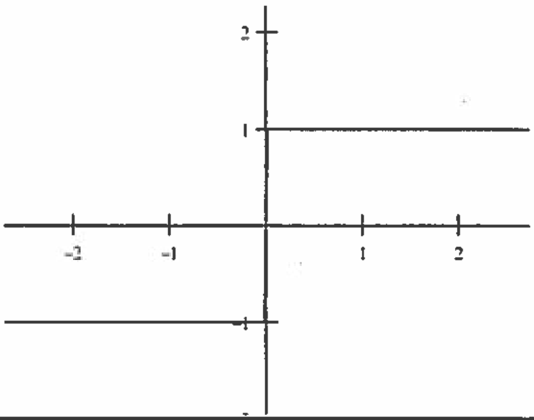


$$f(x)=\begin{cases}1, x\neq 2\\ 0, x=2\end{cases}$$

Answer: $\lim_{x\to 2} f(x) =$ 1

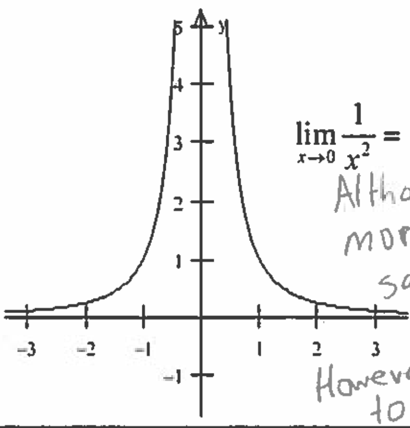
Limits that Do Not Exist

1) The left and right limits are different.



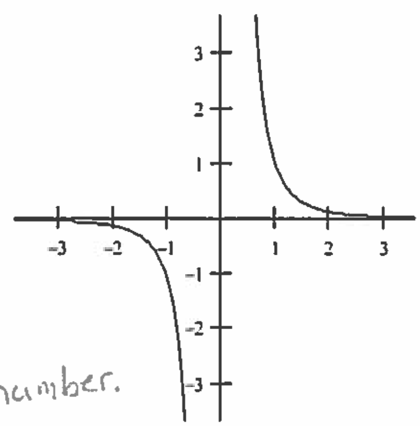
$\lim_{x\to 0}\frac{|x|}{x} =$ DNE

2) Unbounded Behavior



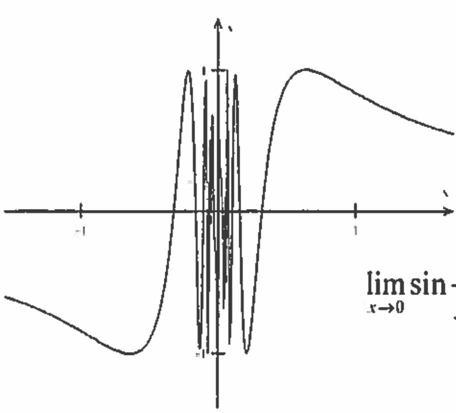
$\lim_{x\to 0}\frac{1}{x^2} =$ DNE

Although you could more specifically say it appoaches positive infinity. However, the limit L has to be a constant number.



$\lim_{x\to 0}\frac{1}{x^3} =$ DNE

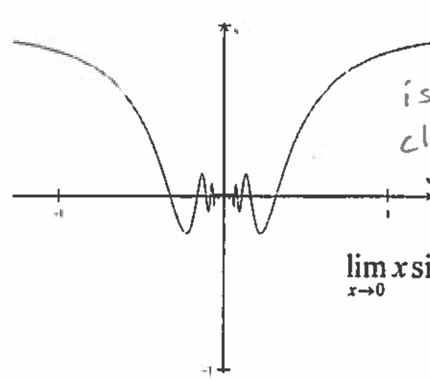
3) Oscillating Behavior



$\lim_{x\to 0}\sin\frac{1}{x} =$

DNE
jumps back and forth between +1 and -1

*Exception to Oscillating Behavior



each oscillation is getting smaller, closer to zero

$\lim_{x\to 0} x\sin\frac{1}{x} = 0$