1.3 Evaluating Limits Analytically

Pg. 67-68 #'s 19-35 odd, 37, 38, 47-53 odd, 196

19) $\lim_{x \to 1} \frac{x}{x^2+4} = \frac{1}{(1)^2+4} = \frac{1}{5}$

21) $\lim_{x \to 7} \frac{3x}{\sqrt{x+2}} = \frac{3(7)}{\sqrt{7+2}} = \frac{21}{3} = 7$

27) $\lim_{x \to \pi/2} \sin x = \sin(\frac{\pi}{2}) = 1$

29) $\lim_{x \to 1} \cos \frac{\pi x}{3} = \cos \frac{\pi}{3} = \frac{1}{2}$

31) $\lim_{x \to 0} \sec 2x = \sec(0) = \frac{1}{\cos(0)} = \frac{1}{1} = 1$

33) $\lim_{x \to \frac{5\pi}{6}} \sin x = \sin(\frac{5\pi}{6}) = \frac{1}{2}$

35) $\lim_{x \to 3} \tan(\frac{\pi x}{4}) = \tan \frac{3\pi}{4} = \frac{\sin \frac{3\pi}{4}}{\cos \frac{3\pi}{4}} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

25) $f(x) = 4 - x^2 \quad g(x) = \sqrt{x+1}$

a) $\lim_{x \to 1} f(x)$

$\lim_{x \to 1} 4 - x^2 = 4 - 1$

$= 3$

b) $\lim_{x \to 3} g(x)$

$\lim_{x \to 3} \sqrt{x+1} = \sqrt{3+1}$

$= 2$

c) $\lim_{x \to 1} g(f(x))$

$\lim_{x \to 1} g(f(1)) = g(3) = 2$

23) $f(x) = 5 - x \quad g(x) = x^3$

a) $\lim_{x \to 1} f(x)$

$\lim_{x \to 1} 5 - x = 5 - 1$

$= 4$

b) $\lim_{x \to 4} g(x)$

$\lim_{x \to 4} x^3 = 4^3$

$= 64$

c) $\lim_{x \to 1} g(f(x))$

$\lim_{x \to 1} (5-x)^3 = (5-1)^3 = 4^3$

$= 64$

37) $\lim_{x\to c} f(x) = 3 \quad \lim_{x\to c} g(x) = 2$

a) $\lim_{x\to c} [5g(x)] = 5(2) = 10$

b) $\lim_{x\to c} [f(x)+g(x)] = 3+2 = 5$

c) $\lim_{x\to c} [f(x)g(x)] = 3\cdot 2 = 6$

d) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{3}{2}$

38) $\lim_{x\to c} f(x) = 2 \quad \lim_{x\to c} g(x) = 3/4$

a) $\lim_{x\to c} 4[f(x)] = 4\cdot 2 = 8$

b) $\lim_{x\to c} [f(x)+g(x)] = 2 + \frac{3}{4} = \frac{11}{4}$

c) $\lim_{x\to c} [f(x)g(x)] = 2\cdot 3/4 = 6/4 = 3/2$

d = $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{2}{3/4} = 2\cdot\frac{4}{3} = 8/3$

47) $\lim_{x\to 0} \frac{x}{x^2-x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{x}{x(x-1)} = \frac{1}{x-1} = \frac{1}{-1} = -1$

49) $\lim_{x\to 4} \frac{x-4}{x^2-16} = \frac{0}{0}$

$\lim_{x\to 4} \frac{x-4}{(x-4)(x+4)} = \frac{1}{x+4} = \frac{1}{8}$

51) $\lim_{x\to -3} \frac{x^2+x-6}{x^2-9} = \frac{0}{0}$

$\lim_{x\to -3} \frac{(x+3)(x-2)}{(x+3)(x-3)} = \frac{x-2}{x-3} = \frac{-5}{-6} = \frac{5}{6}$

53) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \frac{0}{0}$

$\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$\lim_{x\to 4} \frac{(x+5)-9}{(x-4)(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

96) An indeterminant form is an expression that cannot be evaluated. $\frac{0}{0}$ is an example.