1.3 Evaluating Limits Analytically

-Evaluate a limit using properties of limits
-Evaluate a limit using dividing and rationalizing techniques

The value of a limit can easily be evaluated by using **direct substitution** if the function is continuous at c (to be discussed in section 1.4).

Thus: $\lim_{x\to c} f(x) = f(c)$

Homework: pages 67-68
#'s 19-35 odd, 37, 38,
47-53 odd, 96

Some basic limits:

1) $\lim_{x\to c} CONSTANT = CONSTANT$ 2) $\lim_{x\to c} x = c$ 3) $\lim_{x\to c} x^n = c^n$

Example)

1) $\lim_{x\to 4} x^3 = 4^3 = 64$ 2) $\lim_{x\to 4} 23 = 23$ 3) $\lim_{x\to 7} x = 7$

Theorem: Properties of Limits
Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits:

$\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = K$

1) Scalar Multiple $\lim_{x\to c}[b \cdot f(x)] = bL$

2) Sum of Diff. $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$

3) Product $\lim_{x\to c}[f(x) \cdot g(x)] = LK$

4) Quotient $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{K}$

5) Power $\lim_{x\to c}[f(x)]^n = L^n$

Example)
Let $\lim_{x\to c} f(x) = P$, $\lim_{x\to c} g(x) = J$, and $\lim_{x\to c} h(x) = 5$

1) $\lim_{x\to c}(f(x) + g(x)) = P + J$ 2) $\lim_{x\to c}[h(x)]^3 = 5^3 = 125$ 3) $\lim_{x\to c} f(x)\frac{g(x)}{h(x)} = \frac{J}{5}$

Limits of Trig Functions

The limits of trigonometric functions work the same way as normal functions. Typically they can be solved by direct substitution and by using limit properties.

Example)

1) $\lim_{x\to 0} \tan(x) = \frac{\sin 0}{\cos 0}$
$= \frac{0}{1}$
$= 0$

2) $\lim_{x\to \pi}(x\cos(x)) = \pi \cos(\pi)$
$= \pi(-1)$
$= -\pi$

3) $\lim_{x\to 0} \sin^2 x = (\sin(0))^2$
$= 0^2$
$= 0$

Finding the Limit of a Rational Function

1) $\lim_{x \to 1} \frac{x^2+x+2}{x+1} = \frac{4}{2} = 2$

2) $\lim_{x \to 1} \frac{x^3-1}{x-1} = \frac{0}{0}$

Bad

Indeterminant form

It cannot be evaluated.

$\lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)}$

$\lim_{x \to 1} x^2+x+1 = 3$

3) $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} = \frac{\sqrt{1}-1}{0} = \frac{0}{0}$ indeterminant

$\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} \cdot \left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$

$\lim_{x \to 0} \frac{(x+1)-1}{x(\sqrt{x+1}+1)}$

$\lim_{x \to 0} \frac{x}{x(\sqrt{x+1}+1)}$

$\lim_{x \to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$