1.4 Intermediate Value Thm, Squeeze Thm, Continuity

Pgs. 80 - 81 #'s 61-69 odd, 87, 95, 98, 101, 102

61) $f(x) = \begin{cases} 3x^2, & x \geq 1 \\ ax-4, & x < 1 \end{cases}$

$f(1) = 3(1)^2 = 3$

$\lim_{x \to 1^-} ax-4 = a-4$

$a-4 = 3$

$a = 7$

63) $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

$f(2) = (2)^3 = 8$

$\lim_{x \to 2^+} ax^2 = 4a$

$4a = 8$

$a = 2$

65) $f(x) = \begin{cases} 2, & x \leq -1 \\ ax+b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

$f(3) = -2$

$\lim_{x \to 3^-} (ax+b) = 3a+b$

$3a + b = -2$

$f(-1) = 2$

$\lim_{x \to -1^+} (ax+b) = -a+b$

$-a+b = 2$

$b = 2+a$

$3a + (2+a) = -2$

$4a + 2 = -2$

$a = -1$

$b = 2 + (-1)$

$b = 1$

67) $f(x) = x^2$ $g(x) = x-1$

$h(x) = f(g(x))$

$h(x) = (x-1)^2$

$h(x)$ is everywhere continuous because $f(x)$ and $g(x)$ are both everywhere continuous

69) $f(x) = \frac{1}{x-6}$ $g(x) = x^2+5$

$h(x) = f(g(x))$

$h(x) = \frac{1}{(x^2+5)-6}$

$h(x) = \frac{1}{x^2-1}$

$h(x)$ is continuous everywhere except $x = -1$ and $x = 1$

87) $f(1) = 37/12$ $f(2) = -8/3$

$f(x)$ is continuous on the closed interval $[1,2]$ with one value being positive and another negative. By the IVT, there must exist a zero between those two points.

95) $f(x) = x^2 + x - 1$

$f(0) = -1$ $\quad$ $f(5) = 29$

Since $f(x)$ is continuous on the closed interval $[0,5]$ and $-1 \le 11 \le 29$, there exists a value c, $0 \le c \le 5$, where $f(c) = 11$ by the IVT.

$11 = x^2 + x - 1$

$x = 3$

98) $f(x) = \dfrac{x^2+x}{x-1}$

$f(\frac{5}{2}) = 5.8\overline{3}$ $\quad$ $f(4) = 6.\overline{6}$

$f(x)$ is continuous on the closed inte $[\frac{5}{2}, 4]$ and $5.8\overline{3} \le 6 \le 6.\overline{6}$. Therefore, there exists a value c, $\frac{5}{2} \le c \le 4$ such that $f(c) = 6$ by the IVT.

$6 = \dfrac{x^2+x}{x-1}$

$6x - 6 = x^2 + x$

$x^2 - 5x + 6$ $\quad$ $x = 3$

$(x-3)(x-2)$

101) $f+g$ is always continuous

$\frac{f}{g}$ will not by continuous where $g = 0$

$f(x) = x^2 + 1$ $\quad$ $g(x) = x - 1$

$\frac{f}{g}$ is discontinuous when $x = 1$

102) A removable discontinuity can be filled-in by a well placed value of $f(x)$.

A non-removable discontinuity is where $\lim\limits_{x \to c^-} f(x) \ne \lim\limits_{x \to c^+} f(x)$.

a) $f(x) = \dfrac{1}{x-4}$

b) $f(x) = \dfrac{(x+4)}{(x+4)(x)}$

c) $f(x) = \dfrac{x+4}{(x+4)(x-4)}$