Calculus 1.4 Intermediate Value Thm, Squeeze Thm, Continuity

-Understand and use the Intermediate Value Theorem
-Understand the Squeeze Theorem
-Use properties of continuity to make an interval continuous

Homework: pages 80-81
#'s 61-69 odd, 87, 95, 98, 101, 102

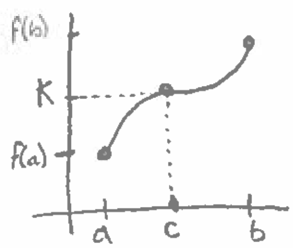
Properties of Continuity

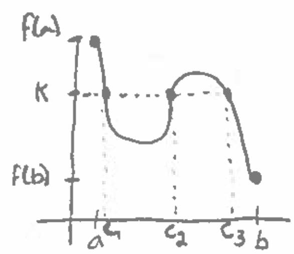
If the functions *f* and *g* are continuous at x=c, then the following functions are also continuous at c:

1) Scalar multiple: $b \cdot f$
2) Sum and difference: $f \pm g$
3) Product: fg
4) Quotient: $\frac{f}{g}$
5) Composition: f o g

The Intermediate Value Theorem (IVT)

If *f* is continuous on the closed interval [a,b] and *k* is any number between f(a) and f(b), then there is <u>at least one</u> number *c* in [a,b] such that *f*(c) = *k*.





Example) Verify the IVT applies to the indicated interval and find the value of c guaranteed by the theorem.

$f(x) = x^2 - 6x + 8$, [0, 3], f(c) = 0

$f(0) = 8 \quad f(3) = -1$

Because f(x) is continuous on the closed interval [0, 3] and $-1 \le 0 \le 8$, then there exists c, $0 \le c \le 3$, where $f(c) = 0$ by the IVT.

$0 = x^2 - 6x + 8$
$0 = (x-4)(x-2)$
$x = 4$ or $x = 2$
$c = 2$

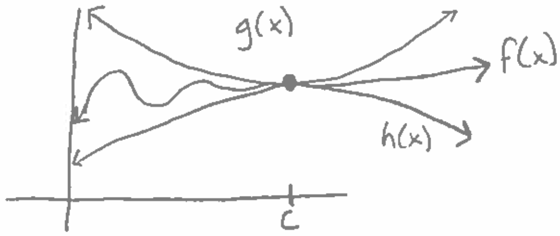
Why do we care about the Intermediate Value Theorem?

If we know that a function is continuous and this function has positive values of f(x) and negative values of f(x), then we can conclude that somewhere $f(x) = 0$.

Squeeze Theorem

If h(x) ≤ f(x) ≤ g(x) for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$,

then $\lim_{x \to c} f(x)$ exists and is equal to L.



Making Piecewise Functions Continuous

For a piecewise function to be continuous each function must be continuous on its specified interval and the limit of the endpoints of each interval must be equal.

Example)
What value of k will make the given piecewise function f(x) continuous?

$$f(x) = \begin{cases} \sin 2x, & x \le \pi \\ 2x + k, & x > \pi \end{cases}$$

$f(\pi) = \sin(2\pi)$

$f(\pi) = 0$

$\lim_{x \to \pi^+} 2x + K = 2\pi + K$

$2(\pi) + K = 0$

$K = -2\pi$

Example)
For what value of k is the function $f(x) = \begin{cases} \dfrac{2x^2 + 5x - 3}{x^2 - 9}, & x \neq -3 \\ k, & x = -3 \end{cases}$ continuous at x = -3?

$$\frac{(2x-1)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \frac{2x-1}{x-3}$$

$$\lim_{x \to -3} \frac{2x-1}{x-3} = \frac{-7}{-6} = 7/6$$