Calculus Section 1.5/3.5 Infinite Limits and Limits at Infinity

- Determine vertical asymptotes
- Properties of Infinite Limits
- Determine horizontal asymptotes

Homework: pg 88 #'s 5, 7, 29, 30, 33-39 odd, 53, 55, 60
pg 202 #'s 17-37 odd, 53, 54

A limit in which f(x) increases or decreases without bound as x approaches c is called an infinite limit.

Infinite limits, or $\lim_{x \to c} f(x) = \pm\infty$, **do not exist**. We just say that the limit equals infinity so we can be more accurate in describing how the limit doesn't exist. This is because a limit must equal a constant number.

Determining Vertical Asymptotes denominator = 0

a. $f(x) = \dfrac{1}{2(x-1)}$

$2(x-1) = 0$ when $x = 1$

$\lim_{x \to 1^+} \dfrac{1}{2(x-1)}$ $\quad \dfrac{1}{2(1.0001-1)}$ $\quad \dfrac{1}{2(+\#)} = \infty$

$\lim_{x \to 1^-} \dfrac{1}{2(x-1)}$ $\quad \dfrac{1}{2(.99999-1)}$ $\quad \dfrac{1}{2(-\#)} = -\infty$

b. $f(x) = \dfrac{2}{\sin x}$

$\sin x = 0$ when $x = 0, \pi, 2\pi, \ldots$

c. $f(x) = \dfrac{x^2+2x-8}{x^2-4}$

$f(x) = \dfrac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$

$f(x) = \dfrac{x+4}{x+2}$

vertical asymptote when $x = -2$

hole/removable discontinuity when $x = 2$

Properties of Infinite Limits

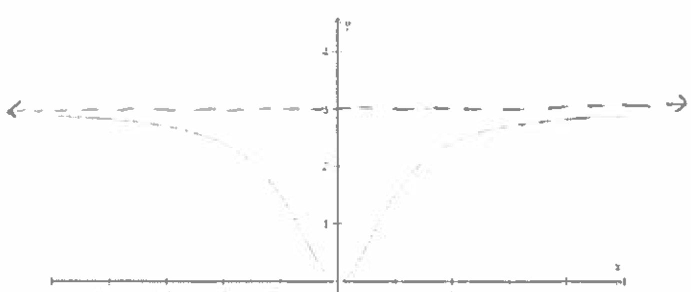
Let $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$

1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty \pm L = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty$ if L > 0

   $\lim_{x \to c} [f(x)g(x)] = -\infty$ if L < 0
3. Quotient $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\infty}{L} = \infty$ $\qquad \lim_{x \to c} \dfrac{g(x)}{f(x)} = \dfrac{L}{\infty} = 0$

Limits at Infinity

Let $f(x) = \dfrac{3x}{x^2+1}$



doesn't always have to be the same

$\lim_{x \to -\infty} f(x) = 3$

$\lim_{x \to \infty} f(x) = 3$

x	$-\infty$	-100	-10	-1	0	1	10	100	∞
f(x)	3	2.999	2.97	1.5	0	1.5	2.97	2.999	3

Finding the limit of a function at $-\infty$ or at ∞ finds the "end behavior" of a function.

Theorem Limits at Infinity

If r is a positive rational number and c is any real number, then

$$\lim_{x\to\infty} \frac{c}{x^r} = 0 = \lim_{x\to -\infty} \frac{c}{x^r}$$

num. bigger : $\pm\infty$

denom. bigger : 0

same power : ratio of leading coefficients

Examples)

a. $\lim_{x\to\infty} \frac{2x-1}{x+1}$

$$\lim_{x\to\infty} \frac{\frac{2x}{x} - \frac{1}{x}}{\frac{x}{x} + \frac{1}{x}}$$

$$\lim_{x\to\infty} \frac{2 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{2-0}{1+0}$$

$$= 2$$

b. $\lim_{x\to\infty} \frac{3x+5}{4x^2+1}$

$$\lim_{x\to\infty} \frac{\frac{3x}{x^2} + \frac{5}{x^2}}{\frac{4x^2}{x^2} + \frac{1}{x^2}}$$

$$\lim_{x\to\infty} \frac{\frac{3}{x} + \frac{5}{x^2}}{4 + \frac{1}{x^2}} = \frac{0+0}{4+0}$$

$$= 0$$

c. $\lim_{x\to -\infty} \frac{5x^4 - 2x^2}{4x^2+1}$

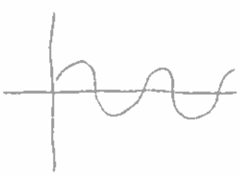
$$\lim_{x\to -\infty} \frac{\frac{5x^4}{x^3} - \frac{2x^2}{x^3}}{\frac{4x^2}{x^3} + \frac{1}{x^3}}$$

$$\lim_{x\to -\infty} \frac{5x - \frac{2}{x}}{\frac{4}{x} + \frac{1}{x^3}} = \frac{-\infty - 0}{0+0}$$

$$= -\infty$$

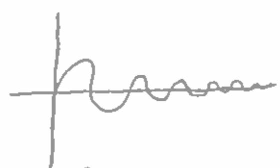
Infinite Limits with Trig Functions

a. $\lim_{x\to\infty} \sin x$ oscillates between -1 and $+1$



we cannot know for certain what the function is as $x\to\infty$ so $\lim_{x\to\infty} \sin x$ DNE b/c of oscillating behavior

b. $\lim_{x\to\infty} \frac{\sin x}{x}$ oscillates but gets smaller



The squeeze thm using $1/x$ and $-1/x$ proves $\lim_{x\to\infty} \frac{\sin x}{x} = 0$

Graph and determine each value for e^x and ln(x).

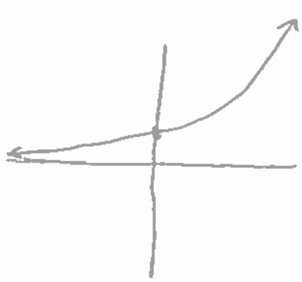
$f(x) = e^x$

$f(0) = e^0 = 1$

$f(1) = e^1 = e \approx 2.718$

$\lim_{x\to\infty} f(x) = e^{\infty} = \infty$

$\lim_{x\to -\infty} f(x) = e^{-\infty} = 0$



$g(x) = \ln(x)$

$g(0) = \ln(0)$ DNE

$g(1) = \ln(1) = 0$

$\lim_{x\to\infty} g(x) = \ln(\infty) = \infty$ very slowly

$\lim_{x\to 0^+} g(x) = -\infty$