10.3 Parametric Equations and Calculus

Pg. 711 #'s 5-11 odd, 15 (1^{st} pt. only), 29, 32, 45-47

5) $x=4t \quad y=3t-2 \quad t=3$

$\frac{dx}{dt}=4 \quad \frac{dy}{dt}=3$

$\frac{dx}{dy}=\frac{3}{4}$ slope: $\frac{3}{4}$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left[\frac{3}{4}\right]}{dx/dt}$

$\frac{d^2y}{dx^2}=\frac{0}{4}=0$ concavity: N/A

7) $x=t+1 \quad y=t^2+3t \quad t=-1$

$\frac{dx}{dt}=1 \quad \frac{dy}{dt}=2t+3$

$\frac{dy}{dx}=\frac{2t+3}{1}=2t+3$ slope: $2(-1)+3=1$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}[2t+3]}{dx/dt}$

$\frac{d^2y}{dx^2}=\frac{2}{1}=2$ concavity: concave up

9) $x=4\cos\theta \quad y=4\sin\theta \quad \theta=\frac{\pi}{4}$

$\frac{dx}{d\theta}=-4\sin\theta \quad \frac{dy}{d\theta}=4\cos\theta$

$\frac{dy}{dx}=\frac{4\cos\theta}{-4\sin\theta}=-\cot\theta$ slope: $-\cot\left(\frac{\pi}{4}\right)=-1$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{d\theta}[-\cot\theta]}{dx/d\theta}$

$\frac{d^2y}{dx^2}=\frac{\csc^2\theta}{-4\sin\theta}$

$\frac{d^2y}{dx^2}=-\frac{1}{4}\csc^3\theta$ concavity: concave down

$\frac{-1}{4\left(\sin^3\left(\frac{\pi}{4}\right)\right)}$ positive

11) $x=2+\sec\theta \quad y=1+2\tan\theta \quad \theta=\frac{\pi}{6}$

$\frac{dx}{d\theta}=\sec\theta\tan\theta \quad \frac{dy}{d\theta}=2\sec^2\theta$

$\frac{dy}{dx}=\frac{2\sec^2\theta}{\sec\theta\tan\theta}=\frac{2\sec\theta}{\tan\theta}=\frac{2\left(\frac{1}{\cos\theta}\right)}{\sin\theta/\cos\theta}=\frac{2}{\sin\theta}$

$\frac{dy}{dx}=2\csc\theta$ slope: $\frac{2}{\sin(\pi/6)}=\frac{2}{1/2}=4$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{d\theta}[2\csc\theta]}{dx/d\theta}=\frac{-2\csc\theta\cot\theta}{\sec\theta\tan\theta}=\frac{-2\cos\theta\cot\theta}{\sin\theta\tan\theta}$

$\frac{d^2y}{dx^2}=-2\cot^3\theta$ concavity: concave down

$-2\left(\cot^3\left(\frac{\pi}{6}\right)\right)$ positive

15) $x = 2\cot\theta \quad y = 2\sin^2\theta \quad \left(\frac{-2}{\sqrt{3}}, \frac{3}{2}\right)$

$y = 2\sin^2\theta$

$\frac{3}{2} = 2\sin^2\theta$

$\frac{3}{4} = \sin^2\theta$

$\frac{\sqrt{3}}{2} = \sin\theta$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = 2\cot\theta$

$\frac{-2}{\sqrt{3}} = 2\cot\theta$

$\frac{-1}{\sqrt{3}} = \cot\theta$

$\tan\theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

$\theta = \frac{2\pi}{3}$

$\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

$\frac{dx}{d\theta} = -2\csc^2\theta$

$\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{-2\csc^2\theta}$

$\frac{dy}{dx} = -2\sin^3\theta\cos\theta$

$\frac{dy}{dx} = -2\sin^3\left(\frac{2\pi}{3}\right)\cos\left(\frac{2\pi}{3}\right)$

$\frac{dy}{dx} = -2\left(\frac{\sqrt{3}}{2}\right)^3\left(-\frac{1}{2}\right)$

$\frac{dy}{dx} = \frac{3\sqrt{3}}{8}$

$y - \frac{3}{2} = \frac{3\sqrt{3}}{8}\left(x + \frac{2}{\sqrt{3}}\right)$

29) $x = 4 - t \quad y = t^2$

$\frac{dx}{dt} = -1 \qquad \frac{dy}{dt} = 2t$

$0 = 2t$

$t = 0$

Vertical tangent:	Horizontal tangent:
None	(4, 0)

32) $x = t^2 - t + 2 \quad y = t^3 - 3t$

$\frac{dx}{dt} = 2t - 1 \qquad \frac{dy}{dt} = 3t^2 - 3$

$0 = 2t - 1 \qquad 0 = 3t^2 - 3$

$t = 1/2 \qquad t = 1$ and $t = -1$

Vertical tangent:	Horizontal tangent:
(1.75, -1.375)	(2, -2) and (4, 2)

45) $x = 3t + 5 \quad y = 7 - 2t$

$\frac{dx}{dt} = 3 \qquad \frac{dy}{dt} = -2$

$S = \int_{-1}^{3} \sqrt{(3)^2 + (-2)^2}\, dt$

$S = 14.422$

46) $x = 6t^2 \quad y = 2t^3$

$\frac{dx}{dt} = 12t \qquad \frac{dy}{dt} = 6t^2$

$S = \int_{1}^{4} \sqrt{(12t)^2 + (6t^2)^2}\, dt$

$S = 156.525$

47) $x = e^{-t}\cos t \quad y = e^{-t}\sin t$

$\frac{dx}{dt} = e^{-t}(-\sin t) + \cos t\,(-e^{-t})$

$\frac{dy}{dt} = e^{-t}(\cos t) + \sin t\,(-e^{-t})$

$S = \int_{0}^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$S = 1.120$