Section 10.4 Polar Equations and Graphs

1) $y=4$

$r\sin\theta = 4$

$\boxed{r = \frac{4}{\sin\theta}}$

2) $3x-5y+2=0$

$3r\cos\theta - 5r\sin\theta = -2$

$r(3\cos\theta - 5\sin\theta) = -2$

$\boxed{r = \frac{-2}{(3\cos\theta - 5\sin\theta)}}$

3) $x^2+y^2=25$

$r^2 = 25$

$\boxed{r = \pm 5}$

4) $r = \frac{3}{\cos\theta}$

$r\cos\theta = 3$

$\boxed{x = 3}$

5) $r = 2\sin\theta$

$r^2 = 2r\sin\theta$

$x^2+y^2 = 2y$

$y^2 - 2y + \underline{(-1)^2} + x^2 = 0 + \underline{(-1)^2}$

$\boxed{x^2 + (y-1)^2 = 1}$

6) $\theta = \frac{5\pi}{6}$

$\tan\theta = \tan\frac{5\pi}{6}$

$\frac{y}{x} = \frac{1/2}{-\sqrt{3}/2}$

$\frac{y}{x} = \frac{1}{-\sqrt{3}}$

$\frac{y}{x} = \frac{-\sqrt{3}}{3}$

$\boxed{y = \frac{-\sqrt{3}x}{3}}$

7) $r = 2+3\sin\theta$, $\theta = \frac{3\pi}{2}$

$\frac{dy}{dx} = \frac{(2+3\sin\theta)(\cos\theta) + (\sin\theta)(3\cos\theta)}{(2+3\sin\theta)(-\sin\theta) + (\cos\theta)(3\cos\theta)}$

$\frac{dy}{dx} = \frac{(2+3(-1))(0) + (-1)(3(0))}{(2+3(-1))(-(-1)) + (0)(0)} = \frac{0}{1} = \boxed{0}$

8) $r = 3-3\cos\theta$, $\theta = \frac{\pi}{2}$

$\frac{dy}{dx} = \frac{(3-3\cos\theta)(\cos\theta) + (\sin\theta)(3\sin\theta)}{(3-3\cos\theta)(-\sin\theta) + (\cos\theta)(3\sin\theta)}$

$\frac{dy}{dx} = \frac{(3-0)(0) + (1)(3)}{(3-0)(-1) + (0)(3)} = \frac{3}{-3} = \boxed{-1}$

9) $r = 4\sin\theta$, $\theta = \frac{\pi}{3}$

$\frac{dy}{dx} = \frac{(4\sin\theta)(\cos\theta) + (\sin\theta)(4\cos\theta)}{(4\sin\theta)(-\sin\theta) + (\cos\theta)(4\cos\theta)}$

$\frac{dy}{dx} = \frac{(4(\frac{\sqrt{3}}{2}))(\frac{1}{2}) + (\frac{\sqrt{3}}{2})(4(\frac{1}{2}))}{(4(\frac{\sqrt{3}}{2}))(-\frac{\sqrt{3}}{2}) + (\frac{1}{2})(4(\frac{1}{2}))}$

$\frac{dy}{dx} = \frac{\sqrt{3} + \sqrt{3}}{-3+1} = \frac{2\sqrt{3}}{-2} = \boxed{-\sqrt{3}}$

10) $r = 2\sin(3\theta)$, $\theta = \frac{\pi}{4}$

$\frac{dy}{dx} = \frac{(2\sin(3\theta))(\cos\theta) + (\sin\theta)(6\cos(3\theta))}{(2\sin(3\theta))(-\sin\theta) + (\cos\theta)(6\cos(3\theta))}$

$\frac{dy}{dx} = \frac{(2(\frac{\sqrt{2}}{2}))(\frac{\sqrt{2}}{2}) + (\frac{\sqrt{2}}{2})(6(-\frac{\sqrt{2}}{2}))}{(2(\frac{\sqrt{2}}{2}))(-\frac{\sqrt{2}}{2}) + (\frac{\sqrt{2}}{2})(6(-\frac{\sqrt{2}}{2}))}$

$\frac{dy}{dx} = \frac{1 + -3}{-1 + -3} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$

11) $r = 1 + \sin\theta$

$$\frac{dy}{dx} = \frac{(1+\sin\theta)(\cos\theta) + (\sin\theta)(\cos\theta)}{(1+\sin\theta)(-\sin\theta) + (\cos\theta)(\cos\theta)} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$$

horizontal

$\frac{dy}{d\theta} = \cos\theta + 2\sin\theta\cos\theta = 0$

when $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

$r = 1 + \sin\frac{\pi}{2} = 2$

$r = 1 + \sin\frac{7\pi}{6} = 1/2$

$r = 1 + \sin\frac{11\pi}{6} = 1/2$

$\left(2, \frac{\pi}{2}\right) \left(\frac{1}{2}, \frac{7\pi}{6}\right) \left(\frac{1}{2}, \frac{11\pi}{6}\right)$

vertical

$\frac{dx}{d\theta} = -\sin\theta - \sin^2\theta + \cos^2\theta = 0$

when $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$r = 1 + \sin\frac{\pi}{6} = 3/2$

$r = 1 + \sin\frac{5\pi}{6} = 3/2$

$(3/2, \pi/6)$ $(3/2, 5\pi/6)$

$\left(\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ can't both be $0\right)$