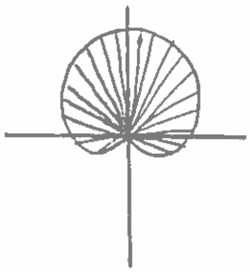
Calculus Section 10.5 Polar Equations and Area

The area bounded by the polar curve $r = f(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is given by the formula:

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta,$$

<u>Ex</u>. Find the area bounded by the graph of $r = 2 + 2\sin\theta$.



$$A = \frac{1}{2}\int_0^{2\pi} (2+2\sin\theta)^2 d\theta$$

$$A = \frac{1}{2}\int_0^{2\pi} (4 + 8\sin\theta + 4\sin^2\theta) d\theta$$

$$A = \int_0^{2\pi} (2 + 4\sin\theta + 2\sin^2\theta) d\theta$$

$$A = \int_0^{2\pi} \left(2 + 4\sin\theta + 2\left(\frac{1-\cos 2\theta}{2}\right)\right) d\theta$$

$$A = \int_0^{2\pi} (2 + 4\sin\theta + 1 - \cos 2\theta) d\theta$$

$$A = \int_0^{2\pi} (3 + 4\sin\theta - \cos 2\theta) d\theta$$

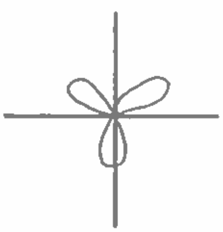
$$A = \left[3\theta - 4\cos\theta - \tfrac{1}{2}\sin 2\theta\right]_0^{2\pi}$$

$$A = (6\pi - 4 - 0) - (0 - 4 - 0)$$

$$A = 6\pi - 4 + 4$$

$$\boxed{A = 6\pi}$$

<u>Ex</u>. Find the area of one petal of $r = 2\sin 3\theta$.



$$2\sin 3\theta = 0$$

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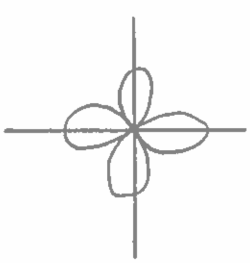
$$3\theta = 0, \pi, 2\pi$$

$$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$A = \frac{1}{2}\int_0^{\pi/3} (2\sin(3\theta))^2$$

$$\boxed{A = 1.047 = \frac{\pi}{3}}$$

<u>Ex</u>. Find the area of one petal of $r = 4\cos 2\theta$.



$$4\cos 2\theta = 0$$

$$\cos 2\theta = 0$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$A = \frac{1}{2}\int_{\pi/4}^{3\pi/4} (4\cos 2\theta)^2 d\theta$$

$$\boxed{A = 6.283 = 2\pi}$$