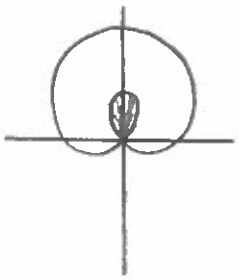
Calculus Section 10.5 Polar Equations and Area Day II

The area bounded by the polar curve $r=f(\theta)$ between $\theta=\alpha$ and $\theta=\beta$ is given by the formula:

$$A=\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta,$$

Ex. Find the area for the inner loop of the graph $r=1+2\sin\theta$



$1+2\sin\theta=0$

$2\sin\theta=-1$

$\sin\theta=\frac{-1}{2}$

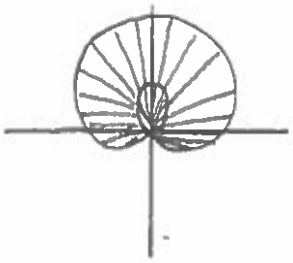
$\theta=\frac{7\pi}{6},\ \frac{11\pi}{6}$

$$A=\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(1+2\sin\theta)^2 d\theta$$

$A=.544$

Ex. Find the area of the graph $r=1+2\sin\theta$

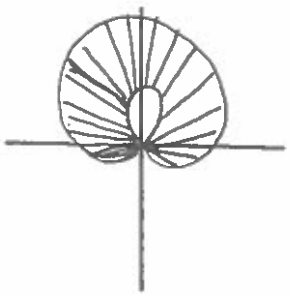
Inner circle is double-counted



$$A=\frac{1}{2}\int_{0}^{2\pi}(1+2\sin\theta)^2 d\theta-\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(1+2\sin\theta)^2 d\theta$$

$A=8.881$

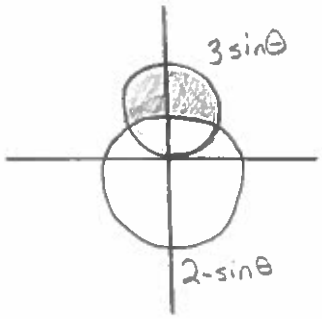
Ex. Find the area between the two loops for the graph $r=1+2\sin\theta$



$$A=\frac{1}{2}\int_{0}^{2\pi}(1+2\sin\theta)^2 d\theta-2\times\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(1+2\sin\theta)^2 d\theta$$

$A=8.338$

Ex. Find the area inside $r = 3\sin\theta$ and outside $r = 2 - \sin\theta$.



$3\sin\theta = 2 - \sin\theta$

$4\sin\theta = 2$

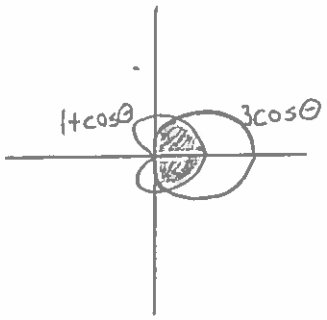
$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3\sin\theta)^2\, d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2-\sin\theta)^2\, d\theta$$

$A = 5.196$

Ex. Find the area of the common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$.



$3\cos\theta = 1 + \cos\theta$

$2\cos\theta = 1$

$\cos\theta = 1/2$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

symmetry: double the top halves

$$A = 2\times\frac{1}{2}\int_{0}^{\frac{\pi}{3}} (1+\cos\theta)^2\, d\theta + 2\times\frac{1}{2}\int_{\pi/3}^{\pi/2} (3\cos\theta)^2\, d\theta$$

$A = 3.927 = \frac{5\pi}{4}$