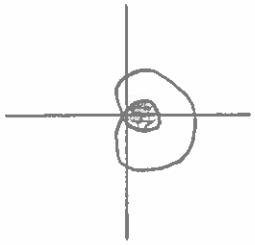
10.5 Polar Area Day 2

1) inner loop of $r = 1 + 2\cos\theta$



$1 + 2\cos\theta = 0$

$2\cos\theta = -1$

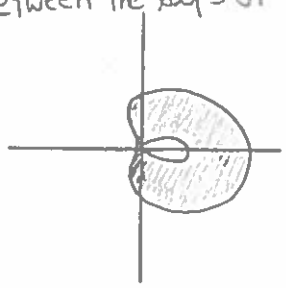
$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 2\cos\theta)^2 \, d\theta$$

$\boxed{A = .544}$

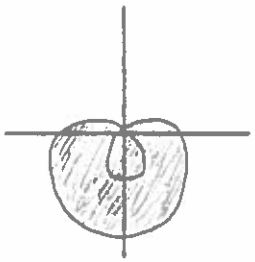
2) between the loops of $r = 1 + 2\cos\theta$



$$A = \frac{1}{2}\int_{0}^{2\pi} (1 + 2\cos\theta)^2 d\theta - \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 2\cos\theta)^2 \, d\theta$$

$\boxed{A = 8.338}$

3) $r = 2 - 4\sin\theta$



$2 - 4\sin\theta = 0$

$-4\sin\theta = -2$

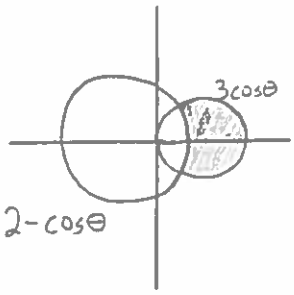
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_{0}^{2\pi} (2 - 4\sin\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (2 - 4\sin\theta)^2 \, d\theta$$

$\boxed{A = 35.525}$

4) inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$



$3\cos\theta = 2 - \cos\theta$

$4\cos\theta = 2$

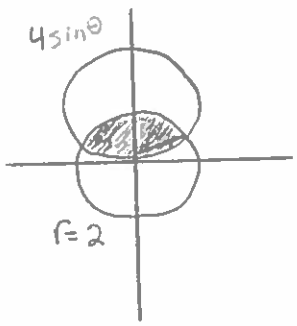
$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$$A = 2 \times \frac{1}{2}\int_{0}^{\pi/3} (3\cos\theta)^2 \, d\theta - 2 \times \frac{1}{2}\int_{0}^{\pi/3} (2 - \cos\theta)^2 \, d\theta$$

$\boxed{A = 5.196}$

5) common interior of $r=4\sin\theta$ and $r=2$



$4\sin\theta = 2$

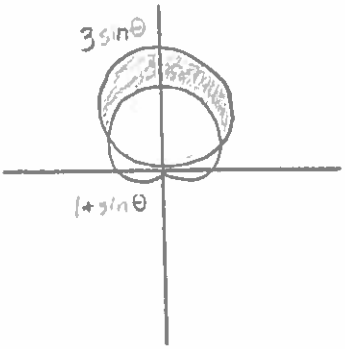
$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2)^2\,d\theta + 2\times\frac{1}{2}\int_{0}^{\pi/6} (4\sin\theta)^2\,d\theta$$

$\boxed{A = 4.913}$

6) inside $r=3\sin\theta$ and outside $r=1+\sin\theta$



$3\sin\theta = 1+\sin\theta$

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2\,d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1+\sin\theta)^2\,d\theta$$

$\boxed{A = 3.142 = \pi}$