Vectors Day 1

1) $x = t^2 - 1 \qquad y = e^{t^3}$

$x'(t) = 2t \qquad y'(t) = e^{t^3} \cdot (3t^2)$

$$\frac{dy}{dx} = \frac{3t^2 e^{t^3}}{2t} = \frac{3te^{t^3}}{2}$$

2) $x(t) = \ln(t^2+5t) \qquad y(t) = 3t^2$

$x'(t) = \frac{1}{t^2+5t}(2t+5) \qquad y'(t) = 6t$

$x'(2) = \frac{9}{14} \qquad y'(2) = 12$

$v(2) = \langle \frac{9}{14}, 12 \rangle$

3) $x(t) = t^5 - 1 \qquad y(t) = 3t^4 - 2t^3$

$x'(t) = 5t^4 \qquad y'(t) = 12t^3 - 6t^2$

$x''(t) = 20t^3 \qquad y''(t) = 36t^2 - 12t$

$x''(1) = 20 \qquad y''(1) = 24$

$a(1) = \langle 20, 24 \rangle$

4) $x(t) = \sin(3t - \frac{\pi}{2}) \qquad y(t) = 3t^2$

$x'(t) = \cos(3t - \frac{\pi}{2})(3) \qquad y'(t) = 6t$

$x'(\frac{\pi}{3}) = 3\cos(\frac{3\pi}{2} - \frac{\pi}{2}) \qquad y'(\frac{\pi}{3}) = 3\pi$

$x'(\frac{\pi}{2}) = -3$

$v(\frac{\pi}{2}) = \langle -3, 3\pi \rangle$

5) $x'(t) = t + 1 \qquad y(t) = \ln(x)$

$x(t) = \frac{1}{2}t^2 + t + C \qquad y(t) = \ln(\frac{1}{2}t^2 + t + 1)$

$x(0) = \frac{1}{2}(0)^2 + (0) + C \qquad y(1) = \ln(\frac{5}{2})$

$1 = C$

$x(t) = \frac{1}{2}t^2 + t + 1$

$x(1) = \frac{5}{2}$

$x(1) = \langle \frac{5}{2}, \ln(\frac{5}{2}) \rangle$

6) $x'(t) = 1 + t \qquad y'(t) = t^3$

$x(t) = t + \frac{1}{2}t^2 + C \qquad y(t) = \frac{1}{4}t^4 + C$

$x(0) = (0) + \frac{1}{2}(0)^2 + C \qquad y(0) = \frac{1}{4}(0)^4 + C$

$5 = C \qquad 0 = C$

$x(t) = \frac{1}{2}t^2 + t + 5 \qquad y(t) = \frac{1}{4}t^4$

$x(2) = 9 \qquad y(2) = 4$

$x(2) = \langle 9, 4 \rangle$

7) $xy = 10$

$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$

$2(3) + 5\frac{dx}{dt} = 0$

$6 + 5\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = \frac{-6}{5}$$

8) $x(t) = t^3 - \frac{3}{2}t^2 - 18t + 5 \qquad y(t) = t^3 - 6t^2 + 9t + 4$

$x'(t) = 3t^2 - 3t - 18 \qquad y'(t) = 3t^2 - 12t + 9$

$3(t^2 - t - 6) = 0 \qquad 3(t^2 - 4t + 3) = 0$

$3(t-3)(t+2) = 0 \qquad 3(t-3)(t-1) = 0$

$t = 3 \quad t = -2 \qquad t = 3 \quad t = 1$

The particle is at rest at $t = 3$

9) $x(t)=t^3$ $y(t)=t^2-5t+2$

$x'(t)=3t^2$ $y'(t)=2t-5$

$\frac{dy}{dx}=\frac{2t-5}{3t^2}$ at $t=2$

$\frac{dy}{dx}=\frac{-1}{12}$ $\boxed{y+4=-\frac{1}{12}(x-8)}$

10) $x(t)=5t+3\sin t$ $y(t)=(8-t)(1-\cos t)$

$x'(t)=5+3\cos t$ $y'(t)=(8-t)(\sin t)+(1-\cos t)(-1)$

$x(t)=25$ at $t=5.446$

$x'(5.446)=7.008$ $y'(5.446)=-2.228$

$\boxed{v(5.446)=\langle 7.008,-2.228\rangle}$

11) $x(t)=t^2-3$ $y(t)=\frac{2}{3}t^3$

$x'(t)=2t$ $y'(t)=2t^2$

Speed $=\sqrt{(10)^2+(50)^2}$

a) $\boxed{\text{speed}=\sqrt{2600}}$

b) $d=\int_0^5\sqrt{(2t)^2+(2t^2)^2}\,dt$

$\boxed{d=87.716}$

c) $\frac{dy}{dx}=\frac{2t^2}{2t}=t$

$\boxed{\frac{dy}{dx}=\sqrt{x+3}}$

12) $\frac{dx}{dt}=\frac{1}{t+1}$ $\frac{dy}{dt}=2t$

$x(t)=\ln|t+1|+C$ $y(t)=t^2+C$

$x(1)=\ln|2|+C$ $y(1)=1+C$

$\ln 2=\ln 2+C$ $0=1+C$

$0=C$ $-1=C$

a) $\boxed{P(x,y)=\langle \ln|t+1|,t^2-1\rangle}$

b) $x=\ln|t+1|$

$e^x=e^{\ln|t+1|}$

$e^x=t+1$

$t=e^x-1$

$\boxed{y=(e^x-1)^2-1}$

c) $\frac{\Delta y}{\Delta x}$

$\frac{y(4)-y(0)}{x(4)-x(0)}$

$\frac{15-(-1)}{\ln|5|-\ln|1|}$

$\boxed{\frac{16}{\ln 5}}$

$t=1\rightarrow x=\ln|1+1|$

d) $y'=2(e^x-1)e^x$

$y'(\ln 2)=2(e^{\ln 2}-1)e^{\ln 2}$

$y'(\ln 2)=2(2-1)(2)$

$\boxed{y'(\ln 2)=4}$

13) $x=2-3\cos t$ $y=3+2\sin t$

$x'(t)=3\sin t$ $y'(t)=2\cos t$

a) $\boxed{\frac{dy}{dx}=\frac{2\cos t}{3\sin t}}$

b) $\frac{dy}{dx}=\frac{2\cos(\frac{\pi}{4})}{3\sin(\frac{\pi}{4})}=\frac{2}{3}$

$x(\frac{\pi}{4})=2-3\cos(\frac{\pi}{4})=2-\frac{3\sqrt{2}}{2}$

$y(\frac{\pi}{4})=3+2\sin(\frac{\pi}{4})=3+\frac{2\sqrt{2}}{2}$

$\boxed{y-(3+\sqrt{2})=\frac{2}{3}\left(x-(2-\frac{3\sqrt{2}}{2})\right)}$

c) $2-3\cos t=0$

$\cos t=\frac{2}{3}$

$t=-.841$ and $t=.841$

$d=\int_{-.841}^{.841}\sqrt{(3\sin t)^2+(2\cos t)^2}\,dt$

$\boxed{d=3.757}$